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The book is intended for astronomers, geophysicists, geochemists and investigators of the Moon and planets. Ten tables, 61 illustrations, bibliography of 223 citations.

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PREFACE

The conquest of the Moon during the 1950's and 60's is one of /3* the brightest pages in the history of mankind. The main problem — to obtain new information for science — united the work of scientists, the talents of designers and the heroism of the spacecraft crews. The basic scientific goal of the conquest of the Moon was and remains the study of the early history and origin of the Earth and the other planets of the solar system. The solution of this problem would give an enormous contribution to the Earth sciences, since it is still impossible to understand the internal structure of the Earth without knowing its history.

A systematic study of the origin of the Earth as a part of a complex astronomical and geophysical problem was begun in the late 1930's and early 40's at the Institute of Earth Physics AN SSSR under the initiative of Academician O.Yu. Shmidt. His idea on the gradual accumulation of the Earth from small solid particles was then developed and supplemented and a physical mechanical model of the formation and evolution of the Earth, which also agrees with the hypotheses of geochemical evolution, was developed at the Institute on its basis.

The study of the origin of the Moon at the Institute of Earth Physics was begun only at the end of the 1940's, also on the basis of an idea of O.Yu. Shmidt, in this case the idea about the formation of satellites in the vicinity of the growing planets. The development of this model has shown that the formation of satellites close to the planets does not simply copy the formation of the planets close to the Sun, their differences are developed here. In addition to this, the evolution of the Earth-Moon system has features peculiar to this system only. The tremendous amount of new data obtained from direct investigations of the Moon has forced a reconsideration of all the existing concepts of the origin of the Moon, including that developed by us. A whole series of advantages of the model of the formation of the Moon in the vicinity of the Earth has been developed. This has stimulated continuation of the work and the elaboration of

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* Numbers in the margin indicate pagination of original foreign text.

certain stages of the early history of the Moon. The main goal of the investigations has been to develop a theoretical model of the formation of the Moon, which, firstly, would be compatible both with the previously known data about the Moon as well as with factors obtained recently by direct conquest of the Moon; secondly, would conform with the history of the lunar orbit which has experienced significant changes because of tidal friction; thirdly, would be joined to the process of the formation of the Earth. An attempt to construct such a model is made in this book. /4

The book consists of six chapters. Factual data about the Moon is presented in the first chapter, the tidal evolution of the Earth-Moon system is discussed in the second, various points of view on the origin of the Moon are compared in the third, and the model of the formation of the Moon and satellites, developed at the Institute of Earth Physics, is discussed in the last three chapters. The work was carried out in the Planetary Geophysics Group of the Institute, first in the Terrestrial Evolution Section, and then in the Theoretical Physics Section. The author thanks co-workers of both sections for help and consideration.

Chapter 1

DATA ON THE MOON

The Moon was a purely astronomical research object as early as /5
the 1930's, which occupied small groups of scientists at a small number of observatories of the world. In the 1940's, interest grew in the Moon as the celestial object closest to the Earth, which is accessible for study with the help of space technology. The tremendous importance of the Moon for the early history of the solar system was realized at the same time. The Moon became considered as a "cornerstone" of planetary cosmogony almost in the literal sense: it was proposed that primary material, from which the terrestrial planets were formed, would be found on its surface. This caused a sharp upward flight of ground-based investigations of the Moon at first, and then its direct conquest.

This chapter is devoted to a survey of those results, which have been obtained for the Moon as an object of geophysical and geochemical research. But it should be noted first that ground-based methods for investigating the Moon have brought a tremendous amount of data about its properties, in spite of the fact that only different sections of the spectrum of light reflected and reradiated by the Moon have been investigated and the apparent motion of the Moon relative to the stars has been measured. Soviet astronomers have made a great contribution to photometric investigations of the Moon, which have given accurate representation of the microstructure and color of the lunar soil. The figure of the Moon has been studied for many decades in the USSR with the help of ground-based optical methods. A theory of radio emission of the Moon was created in the 1940's and 50's and experimental data were first obtained on the variation in temperature of the lunar soil with depth, which permitted an evaluation of the thermal flux from the interior of the Moon averaged over the entire disc. French astronomers first developed the polarimetric methods for investigating the surfaces of the Moon and the planets and obtained the best ground-based photographs of the Moon. Detailed photographic atlases of the Moon were produced in the USA and radar techniques were developed for studying the mesorelief of the Moon, invisible in the

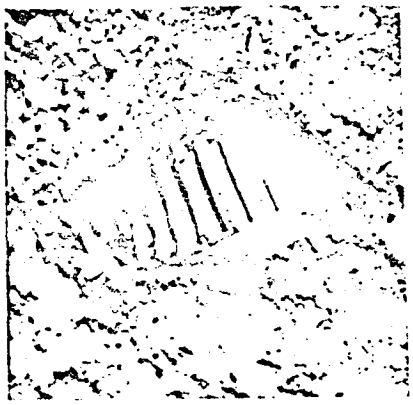

Figure 1.1. Footprint of astronaut in the lunar soil.

telescope. Finally, scientists of various countries have carried out diverse laboratory investigations, which permitted a successful simulation of the lunar soil several years before its conquest. /6

It had been recognized by then that there is no loose dust on the surface of the Moon, but there is a peculiar caked porous layer of low strength, which forms holes with vertical walls when pressed. This layer is called regolith, i.e., cap rock. The term "regolith" is also applied to the surface layer of small stoney satellites, whose surface is found under similar conditions to those of the surface of the Moon.

Figure 1.1 shows the footprint of an astronaut in the lunar regolith, which illustrates the accuracy of the prior model of the lunar soil. The absence of water and atmosphere have resulted in the remarkable preservation of the ancient lunar relief. It has been created both by external factors and internal processes, and lunar geology must accurately take this fact into account.

1.1. Geology of the Moon

The geology of the Moon is the science studying the surface forms of the Moon with the help of the techniques developed for the Earth. The surface of the Moon has been studied for a long time from photographs, where the sharp shadows have permitted evaluation of the height of mountains and circular ridges with an accuracy to several tens of meters. All the formations with horizontal dimensions more than 1 km have been studied in detail from ground-based photographs. It is now well known that the entire Moon is covered with circular maria and craters of all possible dimensions — from the thousand-kilometer craters of Mare Imbrium and Mare Orientalis to microscopic craters. Many traces of the collapse of ridges and filling in of craters by lava are evident in them, but nowhere are there any significant horizontal

shifts, from which one would even suspect "plate tectonics." Folded /7
formations, so typical for the Earth, are also absent on the Moon. Nowhere on the Moon are there sedimentary rocks or any traces of water erosion of the surface.

There is another contrast to the terrestrial surface in that there is no boundary layer on the Moon. A large part of the surface — the elevated highland regions — are composed of rocks enriched with aluminum, calcium and silicon — anorthosites, in whose composition plagioclase predominates. A small part — the depressed parts, the "maria" — are filled with congealed basaltic lava enriched with oxides of iron, magnesium and titanium. This also results in the very dark coloration of the mare basalts, similar in darkness to coal. The Moon is now actually in tectonic rest and in relative rest from external forces. However, in the first billion years both external and internal forces had a very active effect, which led to the extremely complex and peculiar structure of the lunar crust.

The presence of the tremendous number of craters on the Moon has generated an effective method for stydying its surface — crater statistics, which supplements the ordinary terrestrial geological techniques of morphology and stratigraphy. The use of crater statistics for studying the history of the lunar surface must be based on an accurate theory of the origin of the craters. The old controversy between supporters of the volcanic and impact origin of the circular lunar forms can be considered resolved in favor of the impact origin. Let us enumerate the main arguments of this theory.

1. There is a clear regularity in the growth of the number of craters with decreasing dimension, which is described by a power law of the form:

$$dN(a) = \text{const}\, a^{-n} da, \tag{1.1}$$

where a is the radius of the crater, N is the number of craters with radii in the interval from a + da to a — da, and the range in dimensions is about 12 orders of magnitude. This formula reflects the mass distribution of objects falling on the Moon. This regularity is uniform for the maria and for the highlands, but the absolute number of craters of a given size per unit area of the highlands is always approximately

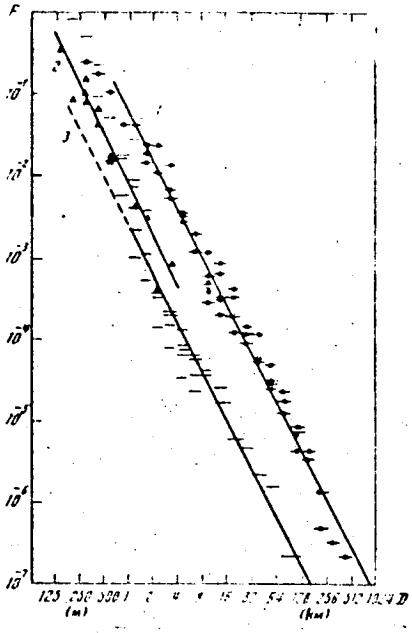

Figure 1.2. Diameter distribution of craters: D - diameter; f - integral number of craters larger than the given dimension per 1 km^2. Craters: 1- highland; 2- "mare period; 3- "post-mare."

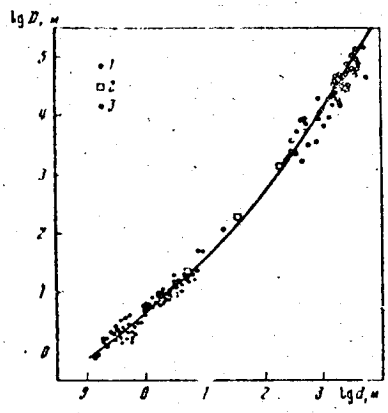

Figure 1.3. Dependence of the depth (d) and diameter (D) for various explosion craters:

1- craters from explosions;
2- meteoritic craters;
3- lunar craters

30 times greater than the corresponding number of craters per unit area of the maria (Figure 1.2) [1]. Such a phenomenon is inexplicable for volcanic processes.

2. The distribution of crater centers on a given uniform surface (mare or highland) is random, if one rejects the grouping of secondary craters and takes into account the obliteration of the previous relief with the fall of larger objects. Moreover, the distribution of volcanoes on a surface is usually coordinated with specific tectonic zones. There are volcanic formations on the Moon - not craters, but hills resembling extinct volcanoes on Earth in form and size. They are located in groups in Mare Imbrium, close to Copernicus and in Mare Nubium [2]. /8

3. The shape of craters is such that the volume of the ridge is approximately equal to the volume of the crater (Schröter's rule). This relation results from the ballistic theory of craters. The depth-diameter relation for lunar craters fits into the unique dependence constructed for terrestrial explosion craters and terrestrial meteoritic craters [3] (Figure 1.3). Neither terrestrial volcanoes nor calderas, i.e., collapsed formations of the volcanic type, have such properties. Moreover, calderas never have such perfectly circular

formations, as if outlined by a circle, as is characteristic of lunar craters.

4. The density of craters on the maria (they are called "post-mare") is such that they correspond roughly to the present spatial density of interplanetary objects reaching the zone of the Earth and capable of providing the required bombardment. Fossil craters on the Earth with ages up to one billion years give a similar distribution /9 frequency per unit area. The contribution of meteoritic material in the lunar regolith, calculated from interplanetary bombardment, has been estimated as approximately 3% [4]. Direct determinations by chemical analysis have given for the meteoritic admixture 1.5 - 2% [5,6]; this can be considered good verification of the theory of meteoritic bombardment of the Moon.

5. The discovery in 1971 of craters of magnitude up to 6 km on Phobos, the satellite of Mars, with the largest diameter of about 25 km, also serves as verification of the impact theory, since the volcanic origin of craters is excluded on such a small object.

The application of crater statistics previously permitted evaluating the relative ages of portions of the lunar surface. Now, after laboratory determinations of the ages of lunar rocks, it has become possible to use the data of crater counts for also determining the absolute ages. The densities of craters per unit area from the data of Hartmann [7] are presented in Table 1 and the determinations of the ages of rocks collected by the Apollo expeditions are presented from the Rb/Sr determinations by Wasserburg and co-workers [8]. The age of rocks for the Apennines are taken from the data of [9], where the $^{40}Ar/^{39}Ar$ method was used. The age of Copernicus is determined indi- /10 rectly as the age of the light material, presumed to belong to the ray of Copernicus. The data of Silver [10] were used.

The dependence of the intensity of crater formation (in units of the present intensity) with time (Figure 1.4) can be obtained from the smoothed data of Table 1. A specific rate of bombardment, which is characterized by a "half-life" $\tau_{1/2}$, is peculiar to each epoch in

TABLE 1

CRATER DENSITY DISTRIBUTIONS F PER UNIT AREA (IN UNITS OF THE AVERAGE DENSITY OF THE POST-MARE CRATERS) AND DETERMINATIONS OF THE AGES t FOR VARIOUS PARTS OF THE LUNAR SURFACE.

Part	F	$t \cdot 10^{-2}$, years	Notes
Tycho	0,06	0,7±0,5	a constant rate of crater formation is assumed
Palus Putreiloinis (Apollo 15)	0,35	3,3±0,06	From one sample
Copernicus	0,5	0,85	Age from Pb isotopes for material of a ray of Copernicus
Mare Fecunditatis (Luna-16)	0,6	3,45	From one sample
Oceanus Procellerum (Apollo-12)	0,75	3,15–3,37	From 9 samples
Mare Tranquillitatis (Apollo-11)	1,6	3,48–3,72	From 7 samples
Fra Mauro (Apollo-14)	2,5	3,85±3,96	From 6 samples
Apennines (Apollo-15)	2,9	4,09±0,19	From 1 sample
Highlands	32	4,2–4,6?	The surface was formed almost at the same time as the Moon

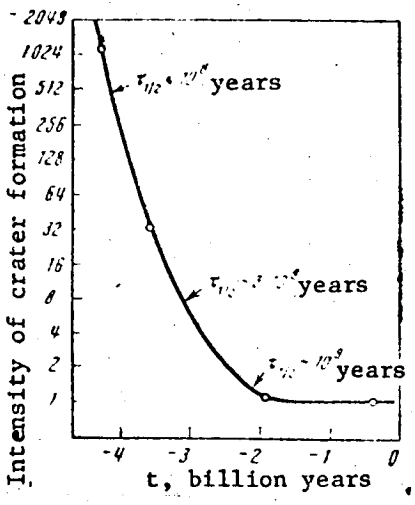

Figure 1.4. History of crater formation on the Moon.

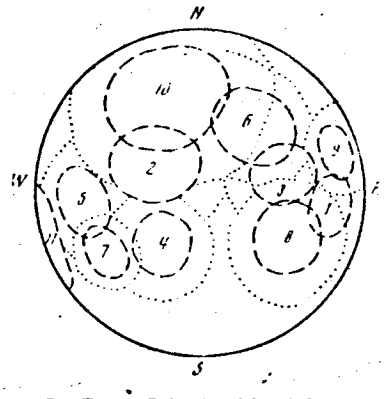

Figure 1.5. Distribution of impact maria and the minimum halos of ejections from them onto the visible surface of the Moon. The numbers give the time sequence of the formations.

the past. By $\tau_{1/2}$ is meant the time for the attenuation of the bombardment intensity by 2 times, i.e., the time for the Moon to scoop up half of the falling objects from their orbits (assumed geocentric). The high rate of bombardment of the lunar surface during the first half billion years of its existence has very much complicated the interpretation of present observational data. In particular, it is very difficult to judge the role of endogenic processes in the highland regions, which were formed before the lunar maria and are composed of different rocks (cf. 1.3).

At the beginning of the conquest of the Moon, when the landing sites of expeditions were still mare sections, a concept was put together about the highlands as a solidified anorthosite "raft", never having molted from the heavier magma [11]. An age was attributed to this "raft" as ancient as the age of the Moon, i.e., $(4.5 - 4.6) \cdot 10^9$ /11
years. However, the hundreds of kilograms of the most ancient anorthositic samples collected by the expeditions to the highland parts were found to be no older than 4 billion years in an overwhelming majority. A large number of such samples were undoubtedly the results of ejections from the interior of the Moon. However, many of them had undergone secondary melting with impact (cf. section 1 in book [12]). It is now clear that if some layers of primary crust of the Moon melted out in the place of the highlands at first, by the present this layered structure could not be preserved in view of the reworking by impact of meteorites to a depth of tens of kilometers. This conclusion is clearly verified by the great abundance of breccias among the samples of highland rock, i.e. conglomerates of varied mineral fragments cemented into lumps. At the end of the period of enhanced bombardment, when an overwhelming majority of the craters on the highlands were already formed, impact excavation of the maria occurred (probably $(3.9 - 3.95) \cdot 10^9$ years ago).

The position of the mare basins on the visible side of the Moon with numbers denoting the sequence of their formation is shown in the diagram presented in Figure 1.5 and the minimum regions covered by the ejections from their craters are outlined [13]. It is possible that

all the remaining surface of the Moon is covered by the ejections with the excavation of the maria; thus, it is extremely difficult to find the "primary" material of the Moon anywhere. All the mountains on the Moon are related in one way or another to ejections from the mare basins or large craters. Thus, they have an embanked origin, completely uncharacteristic of terrestrial mountains.

The role of endogenic processes on the Moon is clearly seen on the maria and in craters with lava-filled bottoms. There is no doubt at all that the mare lava was evolved as a result of the melting of the interior of the Moon and is not a product of the melting of only parts of the surface with impacts of celestial objects, as was proposed by Urey [14]. Even earlier, Kuiper concluded from the higher density of craters on the ejections from the bowl of Mare Imbrium than on the Mare itself that lava filled Mare Imbrium later than its basin was formed [2]. Radioactive dating now confirms this. The time of impact excavation of Mare Imbrium is about $3.9 \cdot 10^9$ years, while the age of its lava in different sections is 3.3 and $3.2 \cdot 10^9$ years [8]. Nowhere on the Moon have been found traces of ejections later than $3.1 \cdot 10^9$ years. This also indicates that the internal life of the Moon died long ago, which is also indicated by the very low seismisity of the Moon, now estimated as at most 10^{-9} of the Earth (cf. below, 1.4).

An important problem of lunar geology is the difficult deciphering of the history of the surface of the Moon at the earliest stage of its existence during the first half billion years, which is also necessary for understanding the early history of the Earth, for which this period has not left any observable evidence on the surface.

1.2. Geochemistry of the Moon /12

The geochemistry of the Moon is also very closely related to space chemistry. Of the material brought back from the Moon, 95% originated somehow or other from basaltic magmas. The composition of these magmas differs greatly from any primary material, which could have evolved from the circumsolar gas-dust cloud. Space chemistry has set the goal of finding the limitations on the composition of the lunar interior,

which follow from analysis of ejected rock collected on the surface.

The six Apollo expeditions and the two automatic stations Luna-16 and Luna-20 have brought back to the Earth samples from 8 points of the lunar surface on the visible hemisphere (cf. Figure on the cover, where the circles denote the landing sites of the Luna stations, the triangles those of the Apollo). The surface of the Moon is evidently more homogeneous in the lithologic respect than the surface of the Earth, for which the collection of samples of all 8 points of the surface would not give an adequate representation of its structure. The high-latitude regions adjacent to the poles and the back side of the Moon have not been investigated at all. However, in spite of this, the available chemical data should be assumed as representative of the composition of the lunar surface as a whole.

Photographic survey of the entire Moon indicates that its entire surface consists of two basic types of crust: highland and mare, where the highlands predominate at the poles and on the back side. The investigation of x-ray fluorescence of the surface, performed on Apollo-15, showed the uniformity of the ratios of Al/Si and Mg/Si for the highlands of the visible and back hemispheres, which indicates their similar chemical composition. The composition of the top lunar layer (regolith) reflects the composition of lower-lying layers. However, an admixture of extra-lunar material is noted in it. Thus, the composition of the lunar interior has been studied mainly from intact pieces of ejected rock.

Table 2, compiled from the data of works [15-17], gives a representation of the basic elemental composition of the lunar material. The content of the 11 most abundant oxides in lunar rocks is presented. The most abundant of them, silica SiO_2, is also the most stable in concentration from sample to sample. Its concentration varies about 40%. Up to now, the unique exception is sample 12013, a small pebble of dimensions 2 X 3 X 4 cm, similar in composition to granite, where the SiO_2 content is about 60% [18]. Iron, aluminum, titanium and magnesium have the most variable concentrations. The calcium content is enhanced in the highland basalts. The variations in content of these elements is greater than in the terrestrial varieties of basalts.

TABLE 2 /13

CHEMICAL COMPOSITION (% BY WEIGHT) OF LUNAR BASALTS AS COMPARED TO THE COMPOSITION OF TERRESTRIAL BASALTS AND METEORITES.

Rocks, region, expedition	SiO_2	TiO_2	Al_2O_3	FeO	MnO	MgO	CaO	Na_2O	K_2O	P_2O_5	Cr_2O_3
MARE BASALTS											
Mare Tranquillitatis (Apollo-11*)	40,7	11,0	9,43	17,42	0,23	7,34	10,52	0,49	0,18	0,12	0,32
Oceanus Procellerum (Apollo-12)	45,0	2,97	9,76	19,7	0,27	10,9	9,97	0,33	0,05	0,09	0,50
Mare Imbrium(Apollo-15)	45,0	2,54	8,9	22,21	0,30	9,08	10,27	0,28	0,03	0,05	—
Mare Serenitatis, Taurus-Littrow (Apollo-17)	37,19	13,14	8,7	19,62	0,28	8,52	10,43	0,32	0,04	0,09	0,42
Mare Fecunditatis (Luna-16)	42,95	5,5	13,88	20,17	0,21	6,05	10,8	0,23	0,16	0,14	0,24
HIGHLAND BASALTS											
Fra Mauro (Apollo-14)	48	1,5	12,0	16	0,29	8,4	12	0,58	0,14	—	0,44
Descartes (Apollo-16)	45,4	0,32	28,63	4,25	0,06	4,38	16,39	0,41	0,06	0,07	—
Taurus-Littrow (Apollo-17)	48,5	0,95	17,2	11,4	—	8,94	11,6	0,40	0,25	—	—
KREEP-BASALTS											
Fra Mauro (Apollo-14)	50	1,3	20	7,7	0,14	8	11	0,63	0,53	—	—
Descartes (Apollo-16)	47,18	1,04	19,98	7,91	0,12	10,34	12,03	0,44	0,32	0,40	—
TERRESTRIAL BASALTS											
Ocean floor basalts	49,15	2,09	15,09	7,58**	0,17	7,75	10,61	2,23	0,3	0,23	—
METEORITES											
Basaltic achondrites	49,0	0,61	11,95	18,05	0,52	9,73	9,03	0,40	0,05	0,11	0,48
Chondrites	38,04	0,11	2,50	12,45	0,25	22,84	1,95	0,98	0,17	0,21	0,36

* From the data of Gast [15].
** Fe_2O_3 content equals 3.35% by weight.

Thus, horizontal chemical inhomogeneities of the Moon are observed within the two basic lithologic types of lunar crust. Comparison with terrestrial basalts indicates that the lunar mare basalts are more like /14 the terrestrial unsaturated (low SiO_2 content) olivine basalts in CaO and Al_2O_3, while the lunar highland rocks with low iron cortent are more like the basalts of the terrestrial ocean ridges. Iron is greater on the whole in lunar basalts than in terrestrial; however, they are mainly represented in the divalent form, excepting only small admixtures of metallic iron, whereas terrestrial basalts always contain significant proportions of trivalent iron (cf. Table 2).

An important charactersitic of the composition of the lunar material is its content of microelements. The first detailed analysis of lunar rocks has already shown significant differences of their

content in lunar and terrestrial basalts (Figure 1.6) [5].

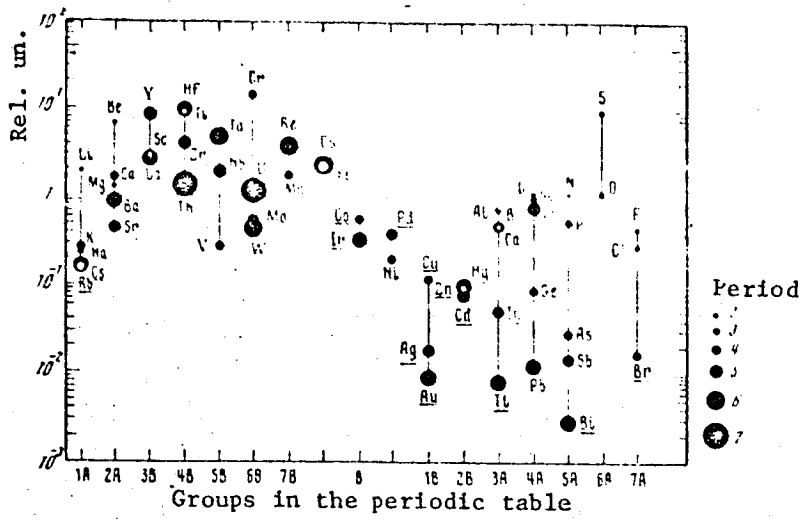

Figure 1.6. Content of the various elements of the periodic table in the basalts of Mare Tranquillitatis with respect to the terrestrial continental basalts.

Gast [15] has divided the microelements into three main categories, which can aid the diagnostics of space chemical processes: siderophylic elements, volatile elements and elements of large ionic radius.

The siderophylic elements — Au, Ce, Ru, Rh, Pd, Re, Os, Ir, Pt, Ni, Co — are the elements more easily reduced than iron and usually accompanying iron in the composition of meteorites. As a rule, they are scarce in lunar basalts as compared to terrestrial basalts, in which in turn they are scarce as compared to chondrites (Figure 1.7). It is significant that the proportions between the contents of all the siderophylic elements in lunar rocks is quite different than in terrestrial rocks. This indicates a difference of geochemical processes in the lunar interior and the processes in the terrestrial mantle. In particular, this fact contradicts the hypotheses of the splitting off of the Moon from the mantle of the Earth after the melting of the terrestrial iron core [19]. One of the scarcer microelements in lunar basalts is gold: it is 10^2 times less than in terrestrial basalts (Figure 1.7). At the same time, it is significantly greater in the lunar regolith. The admixture of gold into the regolith serves as one of

/15

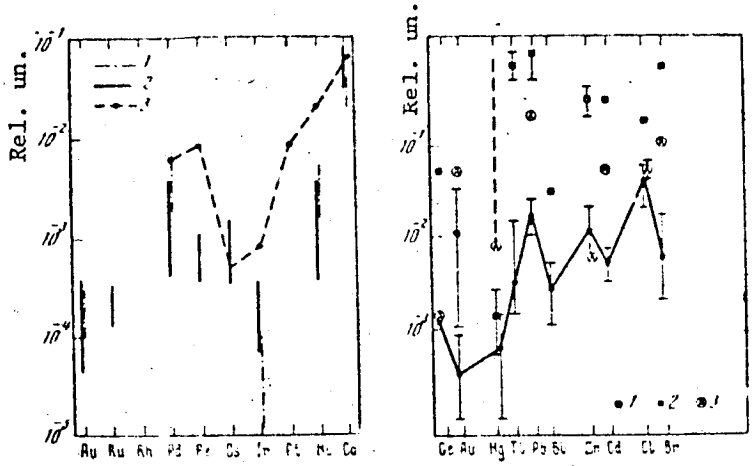

Figure 1.7. Content of the siderophylic elements in lunar (1,2) and terrestrial (3) basalts with respect to the content in chondrites: 1- data of Apollo-11; 2- Apollo-12.

Figure 1.8. Content of volatile elements in terrestrial (1) and lunar (2) basalts and also in basaltic achondrites (3) with respect to the average content for carbonaceous chondrites of the first and second type.

the indications of the introduction of meteoritic material into the surface layer of the Moon [5].

The volatile elements are the group of low-melting elements with melting temperatures below 600°K. These include Pb, Bi, Tl, In, Hg, and Zn, Cd, Cl, Br are also closely adjoining. Almost all these elements are depleted in the lunar basalts by one to three orders of magnitude as compared to terrestrial basalts, which also contain these elements in significantly smaller amounts as compared to carbonaceous chondrites of the first and second type, most enriched in the volatile components (Figure 1.8, and also 1.6). The depletion of the volatile elements in lunar basalts is regularly accompanied by their enrichment with high-melting elements, such as U, Th, Ba and the rare-Earth elements. This property of the ejected rocks of the Moon has paramount importance for understanding the process of the accumulation of the Moon. Since the loss of such heavy elements as Pb, Bi, Tl from /16 the surface of the Moon due to thermal dissipation is negligibly small, then in view of their volatility these elements can only migrate over the lunar surface [20], which does not significantly reflect on their

average contents in the ejected rocks, which remain very low*. Potassium is also a comparatively volatile element. Its content in the Moon is several times less than in the Earth, which appears in the decreased ratio of potassium to uranium.

The systematic depletion of the Moon by all the groups of volatile microelements can be interpreted only as their deficit in the material of the Moon arising up to its accumulation. The mechanism of this depletion must also include the enrichment by the high-melting substances. The procedure for differentiating the pre-lunar material according to volatility, which could be accomplished with the formation of the Moon from the circumterrestrial swarm of satellites, is discussed in Chapter 5.

The lithophylic elements of large ionic radius are K, Rb, Cs, Ba, U, Th and the rare-Earth elements or, as they are abbreviated in the literature, REE. These elements have the properties of being accumulated in the crust. The mare basalts, in which the enhanced content of potassium, the rare-Earth elements and phosphorous were first discovered, were denoted by the abbreviation KREEP from the first letters of these components (cf. Table 2). Later, when it was discovered that uranium and thorium frequently adjoin this group of elements, the term KREEPUTh was sometimes used. The great variation in the content of this group of elements between the mare and highland rocks led to the concept of "KREEP-enriched" (or simple KREEP) and "KREEP-depleted." There is not yet a corresponding term in the Russian language. A sharply reduced content of europium and strontium as compared to the other elements, whose relative enrichment is the order of 10^2, is observed in KREEP-enriched basalts. This is related to the difference in the oxidation-reduction conditions in lunar and terrestrial basalts. On the whole, the enrichment by the lithophylic elements of large ionic radius is correlated with the enrichment by aluminum and the depletion by iron. The Fe/Al ratio increases for KREEP-depleted rocks.

* The migration of lead appears only in the accuracy of the uranium-lead method for determining the ages of lunar rocks. Because of the depletion in the Moon of nonradiogenic lead, almost all the lead on the Moon is of radiogenic origin. The discovery of its migration over the surface has led to the rubidium-strontium and potassium-argon methods becoming preferred as the methods for radioactive dating.

TABLE 3*

CHEMICAL COMPOSITION (% BY WEIGHT) OF ANORTHOSITIC ROCKS OF THE MOON FROM VARIOUS REGIONS

Type of rocks	SiO_2	TiO_2	Al_2O_3	FeO	MnO	MgO	CaO	Na_2O	K_2O	P_2O_5	Cr_2O_3
Lunar anorthosites											
Apollo-11	46,0	0,3	27,3	6,2	0,1	7,9	14,1	0,3	сл.	—	0,2
Apollo-14	43,50	0,19	23,29	4,56	0,06	15,82	12,27	0,28	0,06	0,03	—
Apollo-15	44,08	0,02	36,49	0,29	0,00	0,09	19,68	0,34	0,01	0,01	—
Apollo-16	43,97	0,02	35,83	0,36	—	0,25	18,95	0,34	0,01	—	сл.
Apollo-17	42,7	—	34,9	0,2	—	0,1	19,1	0,01	0,1	0,2	сл.
Luna-16	42,9	0,02	36,0	1,99	0,01	2,52	18,1	0,36	0,02	0,03	0,01
Luna-20	44,2	0,32	19,1	6,91	0,12	13,37	13,3	0,48	0,47	0,17	0,18
Terrestrial anorthosites	54,54	0,52	25,72	1,72	0,02	0,85	9,62	4,66	1,06	0,11	—

NOTE: Data of Table are taken from work [17].

* Translator's note: Commas in numbers represent decimal points.

There is the greatest interest in the problem of the origin of the oldest magmatic rocks: whether they are products of differentiation of large strata of the Moon, metamorphosed with impacts, or whether they first arose as products of melting with intensive crater bombardment. The degree of concentration in the lunar crust of the lithophylic elements of large ionic radius as compared to their concentrations over the whole Moon (if this were known) would give an answer to this problem. The lunar crust composed of anorthosites (Table 3), mare basalts and underlying anorthositic gabbros has a thickness of about 65 km. Judging from seismic velocities, ultrabasic rocks occur at greater depths (cf. below). There was evidently differentiation in depth. This follows mainly from the high concentration of uranium and thorium in the surface layers, which is impossible for the whole Moon, since the Moon would be molten in this case. The other elements exhibit a less sharp concentration, while the degree of their concentration in the crust increases both with the value of the ionic radius as well as with valence. The greater a given element differs from iron and magnesium in these quantities, the greater its concentration in the crust (cf. Table 4 and Figure 1.9, which we have quoted from [13]).

In spite of the described regularities, the chemical composition of the surface of the lunar rocks is very inhomogeneous. Melting in different places of the lunar interior must have been local features,

since the basaltic lavas could not have been products of a single magmatic center. The three main types of magmatic rocks are the following: mare basalts, enriched with iron and darker in color; KREEP-basalts, also enriched with U, Th and Al; anorthosites or rocks enriched with plagioclase — the light rocks of the highlands. They do not have the same enrichment of U, Th and REE, but there is a systematic anticorrelation of Al and Fe + Mg.

TABLE 4*
ELEMENTAL COMPOSITION OF THE MOON AND HIGHLAND CRUST (~10% OF THE VOLUME OF THE MOON)

Element	Content		% in crust with respect to entire Moon
	In Moon	In highland crust	
Si	15,0%	19,7%	13
Ti	0,70	0,41	5,9
Al	12,0	13,2	11
Fe	8,6	4,9	5,7
Mg	8,9	5,12	5,8
Ca	12,8	9,9	7,7
Na (ppm)	1770 ppm	2500 ppm	14,1
K	250	800	32
Cs	0,04	0,1	26
Rb	0,87	2,3	26
Ba	33	144	44
Sr	93	190	20
Eu	0,81	1,7	21
La	3,5	12	34
Y	24	32	13
Th	0,32	1,8	56
U	0,086	0,46	54
Zr	67	156	23
Nb	4,8	10,8	23
Sc	81	7,7	0,95
Mn	700	600	8,6
V	440	25	0,57
Cr	1100	750	8,8

*Translator's note: Commas in numbers represent decimal points.

A comparison of the composition and ages of the main types of ejected rocks leads to the conclusion about the inhomogeneity of the original chemical composition of the Moon with depth. Chondrite material could not have been the basic material from which all the surface rocks of the Moon were extracted [17]. Gast [15] has taken as a working model of the composition of the Moon a certain double-layer model, the top 150 km of which are enriched with plagioclase and depleted in iron and magnesium as compared to chondrites. The composition of the lower-lying strata is as similar as possible to the material of the basaltic chondrites, enriched with high-melting and depleted in volatile

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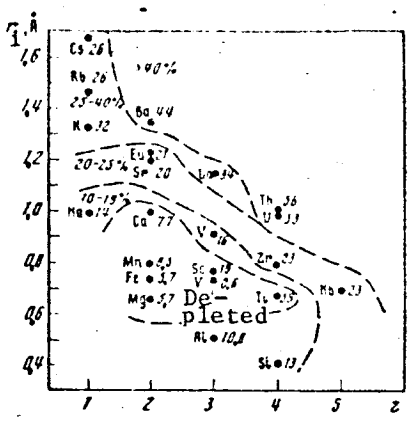


Figure 1.9. Dependence of the enrichment of the lunar crust by various elements (in % with respect to the whole Moon) on the ionic radius r_i and valence z of the element.

TABLE 5**
ASSUMED COMPOSITION (% BY WEIGHT) OF LUNAR INTERIOR.

Oxides	Lower mantle of the Moon	Upper layer, 150 km.	Basaltic achondrites* (Frankford, Govardit).
SiO_2	49,1	48	49,48
TiO_2	0,45	0,8	0,46
Al_2O_3	6,0	16	5,10
FeO	18	9	17,39
Cr_2O_3	0,5		1,34
MgO	19,6	14	20,5
CaO	5,3	12	4,02
Na_2O	0,1	–	0,17
MnO	0,5	0,4	0,-

* Meteorites.

**Translator's note: Commas in numbers represent decimal points.

microelements. The proposed composition of the lunar interior for the nine most abundant oxides is presented in Table 5, taken from work [15].

The representation of the initial vertical inhomogeneity of the lunar depths is also necessary for understanding the chronology of lava eruptions: the rocks enriched with aluminum and calcium erupted first from the upper "story", and then later the mare basalts enriched with iron and depleted in aluminum from the deeper layers. Of course, /19 such a representation includes the hypothesis about the thermal history of the Moon, in which the melting temperature was first reached at a lesser depth, and then at greater depths. We will return to this problem at the end of the first chapter.

1.3. Ages of Lunar Rocks

The rubidium-strontium method is assumed the most reliable method for determining the ages of lunar rocks, although the potassium-argon and uranium-lead methods are also used for comparison [21]. Rubidium is related to the depleted elements in lunar rocks. The radio- /21 genic isotope ^{87}Sr is formed by β-decay with a half life of $(4.7 - 5.0) \cdot 10^{10}$ years. Thus, the ratio $^{87}Sr/^{86}Sr$

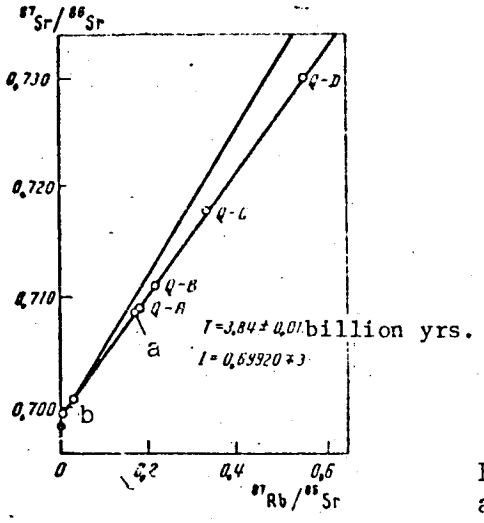
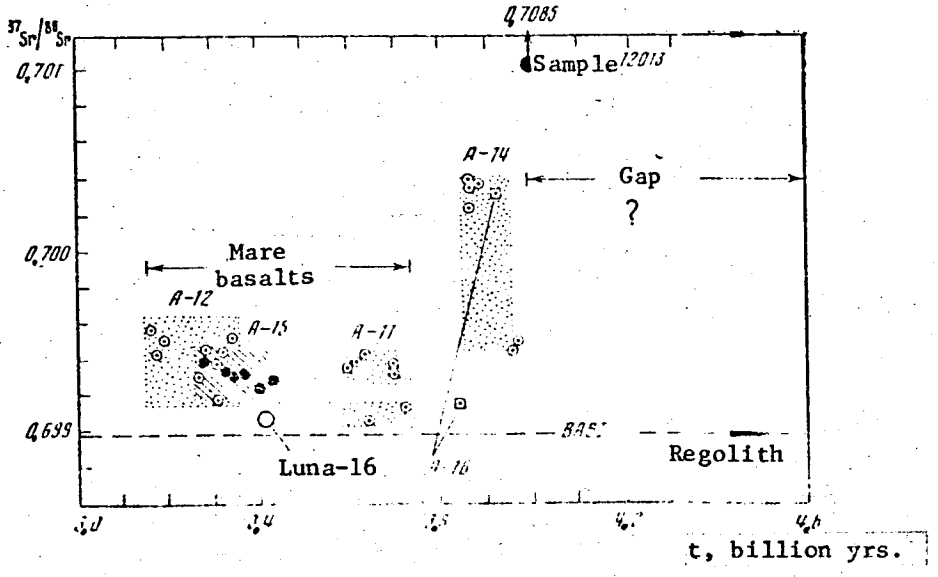


Figure 1.10. Rubidium-strontium isochrone for sample 68415 of the lunar highland rock brought back by Apollo-16. /20

a- Crystallobalite
b- Total plagioclase

Figure 1.11. Composite diagram of the ages of lunar rocks from the rubidium-strontium method.



increases with time, where ^{86}Sr is the nonradiogenic isotope. The rate at which this ratio increases is proportional to the $^{87}Rb/^{86}Sr$ ratio. The lowest value of $^{87}Sr/^{86}Sr$ known for objects of the solar system belongs to the achondrite meteorite Angra dos Reis and equals 0.69884 ± 0.00004. One can make the assumption that this ratio reached to at least such low values in the preplanetary cloud. The $^{87}Sr/^{86}Sr$ ratio is somewhat greater for other achondrites. Its increase by 0.001 corresponds to a difference in age of 10^8 years (the typical ratio for chondrites $^{87}Rb/^{86}Sr = 0.75$). The age of a rock t is calculated

from the formula:

$$t = \frac{1}{\lambda} \ln \left[\frac{(^{87}Sr/^{86}Sr) - (^{87}Sr/^{86}Sr)_0}{^{87}Rb/^{86}Sr} + 1 \right], \qquad (1.2)$$

where λ is the decay constant.

In determining the ages of lunar rocks, all three isotopic ratios were measured for several mineral phases appearing in the sample and also for some "quintessence," enriched with rubidium and strontium, prepared from the entire sample, which permitted constructing a straight line from the points on a graph with axes $^{87}Rb/^{86}Sr$ and $^{87}Sr/^{86}Sr$. The age of the sample is characterized by its slope. Its own initial value of $I = (^{87}Sr/^{86}Sr)_0$, which varies from sample to sample, is obtained for each sample. It is higher than the lowest of the known ratios for basaltic achondrites, which appears in the definitions under the name BABI (Basaltic achondrites best initial). If the ratio of the isotopes can be measured only on the average for a sample (for example, for lunar dust), then, by producing a straight line through the obtained point and through BABI, one obtains the "model age" of the rock, which does not have a precise physical interpretation in individual cases. The average grouped "model age" for a large number of samples can be interpreted under certain assumptions as the age of the Moon. An example of the isochrone constructed from the rubidium-strontium method for sample 68415 is presented in Figure 1.10 from the data of [22]. The authors of this work could measure the age of a basaltic fragment, 62 mg in weight at most, from Mare Fecunditatis brought back by Luna-16 [23]. A cumulative diagram of the ages and initial ratios $(^{87}Sr/^{86}Sr)$, according to Papanastassiou and Wasserburg, is presented in Figure 1.11. It is based on a whole series of painstaking experiments published during 1970-1972 in the journal Earth and Planetary Science Letters; references can be found in [24]. Comparison of these results with the data of other authors shows good agreement: the difference in the ages of rocks does not exceed 10^8 years.

The results of all the numerous experiments in determining the ages of ejected rocks on the Moon is such that almost nowhere are found samples older than $4.0 \cdot 10^9$ years and younger than $3.1 \cdot 10^9$ years. Determination of the age of the Luna-20 samples from the highland region between Mare Crisium and Mare Fecunditatis gave an age of /22

3.95 · 10^9 years [24]. This result was unexpected: it was assumed that rocks as old as the Moon itself, i.e. with an age of about 4.5 · 10^9 years, could be found on the highlands, because the opinion was widely accepted that the highland crust had melted practically simultaneously with the accumulation of the Moon due to the release of gravitational energy. At the same time, it was expected to find traces of vulcanism younger than 3.0 · 10^9 years. As is well known, the age of the so-called "orange" glasses from the landing site of Apollo-17 was 3.7 · 10^9 years, although they could be taken as younger formations from morphological indications [12]. There is a gap in the scale of ages between 4.0 · 10^9 and (4.5 - 4.6) · 10^9 years, which nothing fills. This is evidence about large-scale catastrophes in the history of the lunar surface - the formation of the mare depressions with impacts of large objects coming from geocentric orbits. The formation of the largest of them - the depression of Mare Imbrium - is dated by an age of (3.85 - 3.95) · 10^9 years from the age of breccias from the region of Fra Mauro, the landing site of Apollo-14. All the impact (circular) maria were formed earlier than Mare Imbrium, according to morphological indicators. Mare Orientalis, which appeared later, is the exception. The formation of all the impact maria evidently occupied a comparatively short time interval - at most about 10^8 years, which completed an intensive crater bombardment, whose traces are seen on the highlands [2]. The melting of rocks in the centers of large-scale impacts and their chemical differentiation changed the nature of the distribution of radiogenic isotopes and the products of their decay in the rocks. The dispersion of material from craters over practically the whole surface of the Moon has led to the fact that the "radioactive clocks" for the collected samples reckon time not from the epoch of the accumulation of the Moon or from the epoch of the eruption of the highland crust onto the surface, but from the epoch of the large-scale cataclysms, i.e., events occurring (3.9 - 4.0) · 10^9 years ago.

At the same time, the "model" ages for numerous samples of the regolith, which were obtained for an initial ratio $^{87}Sr/^{86}Sr$ equaling BABI, are grouped in the interval between 4.4 and 4.6 · 10^9 years with a center of about 4 . · 10^9 years (cf. Figure 1.11). It is possible that the time about 4.5 · 10^9 years ago should be considered

the time of accumulation of the Moon, when the original abundances of isotopes were combined.

The determination of the age of sample 68415 and others by the uranium-lead method, for example, serves as an independent verification of the fact that the age of the Moon is about $4.5 \cdot 10^9$ years. The isochrone for this sample, and for other crystalline samples found in the region of Fra Mauro, indicates that the age of the original conglomerates equals $(4.47 \pm 0.02) \cdot 10^9$ years, and the time of their recrystallization is about $3.95 \cdot 10^9$ years ago [24].

The conclusion of Wasserburg that the time of accumulation of the Moon ($4.5 \cdot 10^9$ years ago) does not coincide with the time of formation of meteorites and the Earth ($4.6 \cdot 10^9$ years ago) is of exceptional importance to us. It follows from dynamic considerations (cf. Chapters 4,5) that the difference in ages of the Earth and the Moon is $\sim 10^8$ years. Since this difference in ages is assumed different in the various models of the formation of the Moon, then the estimate of the age of the Moon with the help of radioactive dating of samples has paramount significance for the choice of the correct [illegible]. /23

1.4. Geophysics of the Moon

The figure of the Moon. The contour of the apparent disc of the Moon is best approximated by a circle on which is superimposed the irregularity of the relief. The optical compression of the apparent disc at the poles does not exceed 1:1200 — 1:1300. The elongation of the Moon in the direction toward the Earth is even less. Thus, the reference surface approximating the apparent figure of the Moon is a sphere whose radius is 1738 km. The dynamic figure of the Moon differs from the hydrostatic equilibrium figure; its surface is not level. In the case of hydrostatic equilibrium and for the present equality of the periods of rotation and revolution, the equality:

$$\frac{C-A}{B-A} = \frac{4}{3}, \qquad (1.3)$$

would be valid, where A, B, C are the principal moments of inertia of the Moon [25], where the value of (C - A)/C would have to be equal to 0.0000375 at the present distance of the Moon from the Earth. However,

it has been well known for a long time that the actual value of (C - A)/C is 17 times greater and the value of (B - A)/C is 5 or 6 times greater than the equilibrium values. The hydrostatic nonequilibrium of the figure of the Moon is related to inhomogeneity and indicates the presence of shearing stresses in the mantle of the Moon. The oblateness of the dynamic figure can be explained by a difference in the thickness of the solid shell of the Moon at the poles, where it is greater than at the equator [26, 27]. Then the Moon is isostatically compensated on the global scale. However, the peculiarities of the figure of the Moon have not yet been completely interpreted physically.

The coefficients of expansion of the gravitational field of the Moon in spherical harmonics has been obtained from observations of the orbits of circumlunar satellites. In the so-called L1 model, the first five coefficients are:

$$J_{20} = 2{,}07108 \cdot 10^{-3}, \quad C_{21} = 0{,}34 \cdot 10^{-9},$$
$$J_{30} = 0{,}21 \cdot 10^{-4}, \quad C_{22} = 0{,}02583 \cdot 10^{-4},$$
$$C_{31} = 0{,}20716 \cdot 10^{-4}.$$

However, this model does not take into account all the peculiarities /24 of the lunar field, in particular the presence of mascons, which are vast local positive gravitational anomalies coordinated to the circular (impact) maria [28]. The nature of the mascons and their possible origin are discussed in works [29-32]. The total excess mass producing the mascons is probably $1.5 \cdot 10^{2}$ gm, i.e., about $2 \cdot 10^{-5}$ of the mass of the Moon. We note for comparison that the mass of the objects forming the impact maria is estimated as an order of magnitude greater [2].

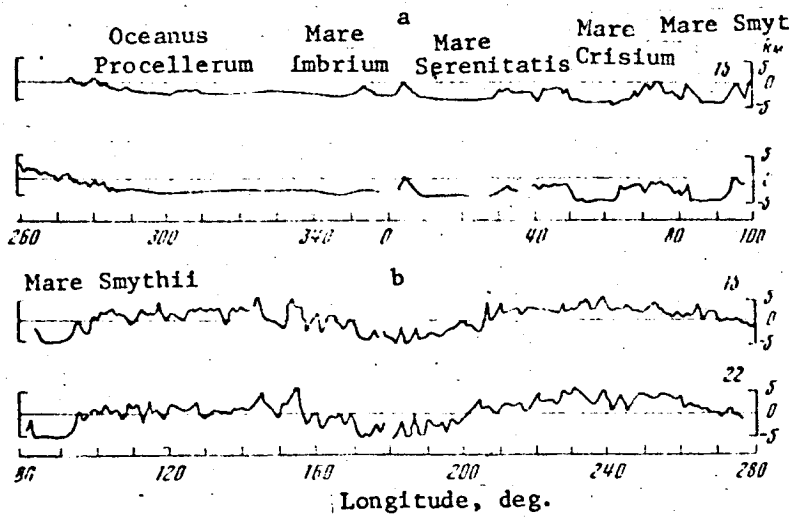


Figure 1.12. Profile of the lunar relief obtained by laser altimetry from Apollo-15. a, b — visible and far sides of the Moon, respectively. Digits above profiles denote orbit number.

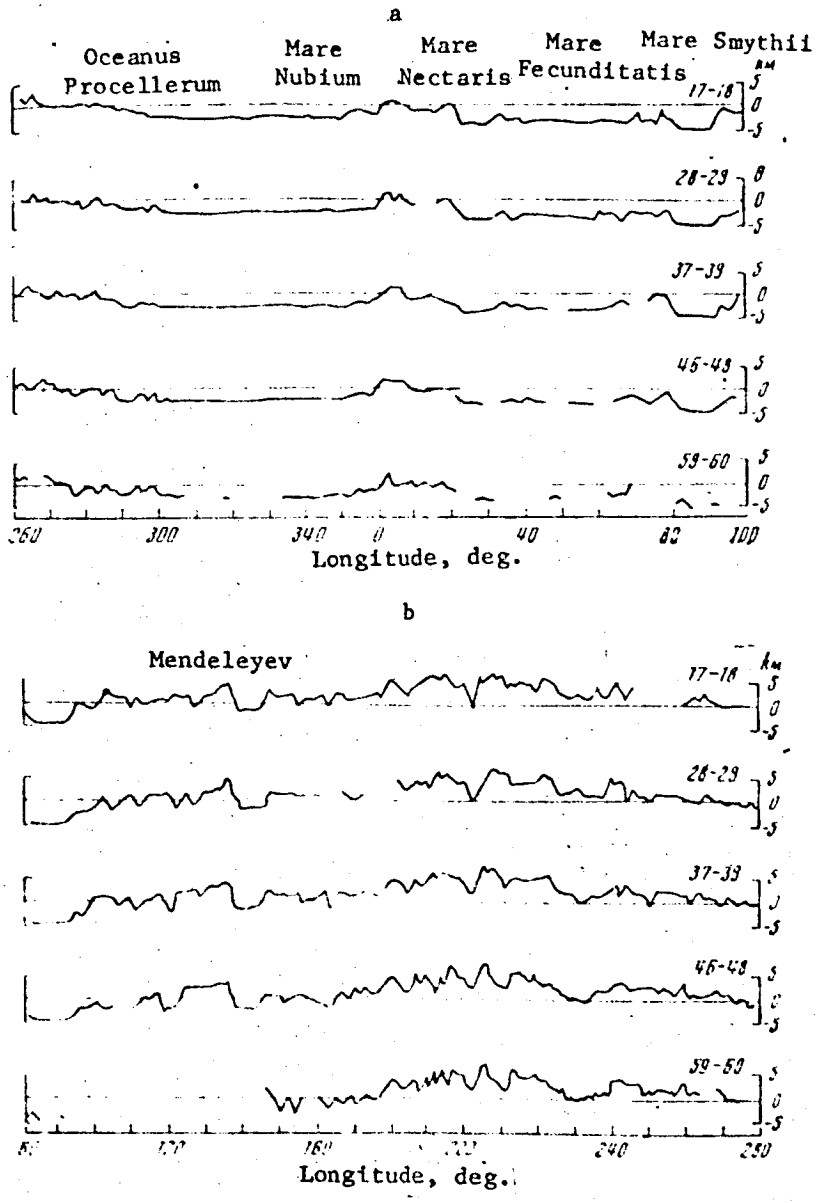


Figure 1.13. Profile of the lunar relief from the data of Apollo-16. Notation is the same as in Figure 1.12.

The most striking peculiarity of the figure of the Moon, discovered recently, is the shift toward the Earth of the center of mass of the Moon with respect to the center of the apparent reference sphere by approximately 3 km. This shift was suspected as early as the flights of the Ranger spacecraft, then repeatedly verified by other data. The results of laser altimetry, carried out from the Apollo-15 and -16 spacecraft [33] with an accuracy of $\pm$ 10 m, which we present below, is very clear evidence of the shift of the center of mass. The magnified

vertical profile of the lunar relief measured with respect to the center of mass of the Moon is presented in Figure 1.12 and 1.13, where one notes a systematic depression of the relief with respect to the apparent reference sphere of radius 1738 km on the near side of the Moon and elevation on the far side. The smoothed altimetry profile /26 with respect to the reference circle is satisfactorily described by the formula:

$$h = +\ 0{,}1 - 2{,}9 \cos(\lambda - 25^\circ) - 1{,}2 \cos 2(\lambda - 50^\circ), \qquad (1.4)$$

where h is the relative altitude in km. The second term on the right in this formula corresponds to the shift of the center of mass of the Moon by approximately 3 km toward the Earth and by 1 km to the left as viewed from the Earth toward the celestial sphere.

The cause of the shift of the center of mass must be a significant inhomogeneity in the distribution of density in the lunar interior. The mascons, which give a shift of the center of gravity toward the Earth, cannot be the main cuase, since the shift caused by them does not exceed 40 m. One can assume that the thickness of the highland crust composed of lighter rocks is greater by 20-30 km on the far side of the Moon than on the near side (cf. Figure 1.14, quoted from [33]). Such an assumption does not contradict the absence of positive gravitational anomalies over the highlands, which rise on the average above the level of the maria by 4 km. It was just noted that the Moon is compensated isostatically on the large scale.

It should be noted that there is no unique "sea level" on the Moon, which would correspond to some equipotential surface, but each mare has its own level and the difference in altitudes of the highlands and maria of 4 km represents an average value. The relief of the maria is significantly smoother than the relief of the highlands. The difference in altitudes on the maria does not usually exceed $\pm$ 150 m, whereas the "jaggedness" of the relief of the highlands reaches several kilometers (cf. Figure 1.12 and 1.13). The origin of the greater thickness of the crust on the far side of the Moon remains unclear. The assumption by Wood [34] about the embanked nature of the thickened crust in connection with the hypothesis of a more intense meteoritic bombardment of the leading hemisphere of the Moon is not completely convincing. It is clear that if the highland crust is a product of

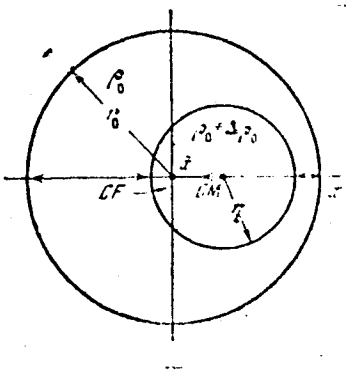


Diagram showing the bulge of the lighter crust (density ρ_0) on the far hemisphere. x- direction toward Earth; CM- center of mass; CF- center of apparent figure.

geochemical differentiation of the lunar mantle, then the large difference in its thickness in the different hemispheres indicates either a spherically asymmetric distribution of the "parent" material of the crust in the Moon or a spherically asymmetric distribution of temperature in the interior, influencing the course of differentiation. A strong inhomogeneity of the structure and composition of the Moon is obtained in both cases. /27

The problem of the presence or absence in the Moon of a dense (iron) central core can be resolved with the help of data on the moment of inertia of the Moon. Up to recent times, several determinations have led to a value of the dimensionless moment of inertia, i.e., $I = C/M_{\mathbb{C}}r_{\mathbb{C}}^2$, greater than 0.4, which is characteristic of a sphere with a homogeneous density. Thus, models of the Moon with a density inversion were constructed. According to recent data [35], $I = 0.395 \pm 0.005$. This indicates that it is not necessary to look for a density inversion in the lunar interior. The present value of the moment of inertia of the Moon permits only a very small mass for an iron core - no more than 1% of the entire mass of the Moon [36].

The liquid or semi-molten core, which the Moon has judging from seismic data (cf. below) and having a radius of about half the radius of the Moon, cannot be iron, if the value of I presented above is correct. Such a core can consist only of molten silicate rocks.

The seismicity of the Moon. At present, several thousand recordings of Moonquakes have been obtained. One part of them is caused by deep seismic shocks, the other by the fall of meteorites onto the Moon. The energy of natural Moonquakes is very small: it is only 10^{-9} of the energy of Earthquakes over identical time intervals. No large Moonquake has been recorded. If the Moon had been heated due to the decay of radioactive elements from an original cold state to the melting temperature of a large part of its interior and then its outer

parts had cooled, then the thermoelastic stresses would have stored an elastic energy capable of causing seismicity by several orders of magnitude greater than that observed [37]. The weak seismicity of the present Moon agrees with its weak tectonics. The seismometers established on the Moon at the landing sites of Apollo-12, -14, -15 and -16 spacecraft are very sensitive. They record micro-Moonquakes, possibly related to thermo-elastic stresses in the regolith arising with the rising and setting of the Sun. Moreover, Moonquakes are recorded, which are coordinated to the tidal cycles of the Moon with periods of 13.6 and 27.2 days and solar tides with a period of 7 months, although the physical relation of the Moonquakes with the tides is still unclear. The very first lunar seismograms were obtained with the fall of ejected rocket stages onto the Moon. They were characterized by a great duration of the ground tremor reaching up to several hours (Figure 1.15). This is related to the small attenuation and at the same time intense scattering of the seismic waves in the upper layer, which is devoid of water and other volatile elements. Active seismic experiments on the surface have indicated that the velocities of seismic waves in the upper 100-meter layer are at most 100 - 300 m/sec, whereas at depths of 5 km they already reach 4 km/sec. /28 If the lunar rocks were as shattered everywhere as at the surface and only compressed by the weight of the overlying layers, then such a great increase of seismic velocities would be impossible [38].

Recording of seismograms with the fall of rocket stages onto specified points of the surface has permitted compiling a seismic section of the Moon, on which is shown the change in the velocities of

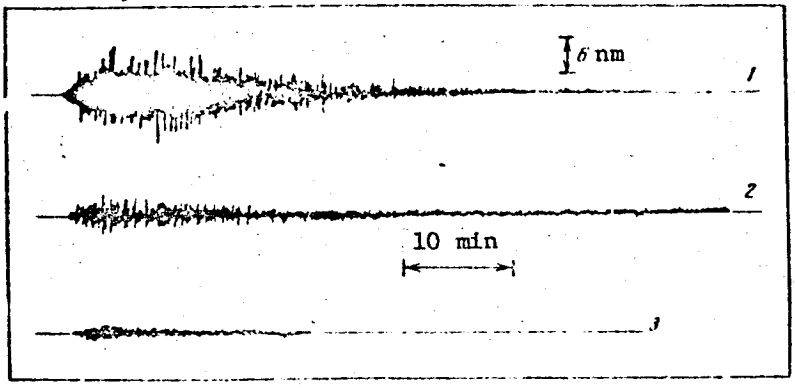


Figure 1.15. Seismograms of Moonquakes. 1- Moonquake caused by the fall of lunar module; 2,3- Natural Moonquakes of December 10 and 16, 1969, respectively.

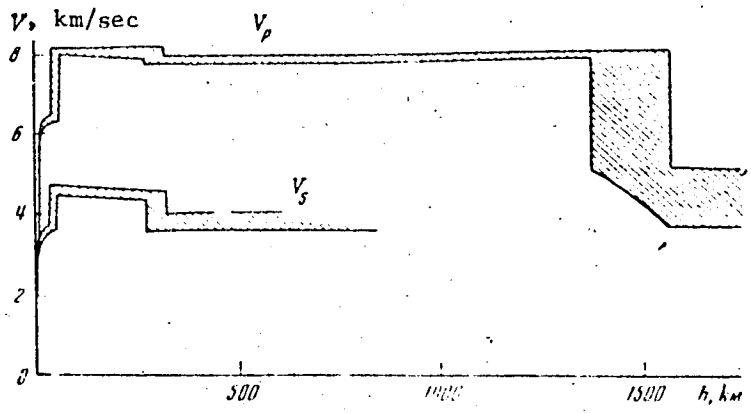


Figure 1.16 Variation of the longitudinal V_p and transverse V_S seismic waves with depth h.

seismic waves with depth [39] (Figure 1.16). The presence of deeply-focussed Moonquakes at depths between 500 and 800 km indicates that the shell of the Moon is not molten to at least these depths. The core of the Moon is evidently semi-molten at greater depths. It is likely that the temperature at the center of the Moon is close to the solidus temperature, which at present is estimated as 1450 - 1460°C.

Analysis of the fall of meteorites has shown that in the mass interval 10^2 - 10^6 gm the flux of meteoritic material can be represented in the form of the dependence:

$$\lg N = -1{,}62 - 1{,}16 \lg m, \qquad (1.5)$$

where N is the integral number of objects with masses greater than m, in gm, which strike 1 km^2 of the lunar surface per year. This flux is one to three orders of magnitude smaller than expected on the basis of calculations of the fall of meteorites on Earth.

A more detailed survey of seismic phenomena and their interpretation are contained in [40]. A diagram of the present structure of the Moon according to the seismic data of [41] is presented in Figure 1.17.

Electromagnetic properties of the Moon. Measurements from circum- /29
lunar satellites and also with the help of magnetometers on the surface of the Moon have shown that the electrical conductivity of the Moon is low. It is 10^{-7} $(ohm\text{-}m)^{-1}$ at the surface and increases to

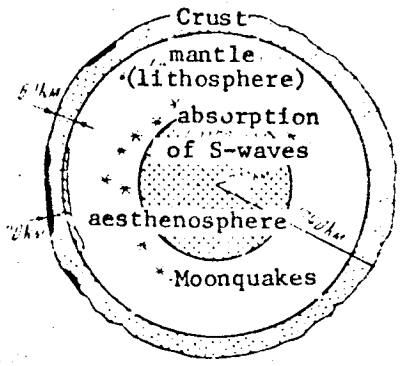


Figure 1.17. Schematic diagram of the structure of the Moon according to data of lunar seismology.

10^{-2} at depths of 600-800 km [42]. Attempts to evaluate the temperature of the lunar interior from these data, by using an analogy to the electrical conductivity of terrestrial rocks, has led to comparatively low values of the temperature of the interior: no greater than 1000 - 1200°C. Such estimates agree poorly with the data on the thermal flux of the Moon and also with the high content of radioactive elements of the Moon (cf. below). At present, data has been obtained that the dependence of the electrical conductivity on temperature for lunar rocks can differ from the terrestrial analogs because of differences in their content of trivalent iron [43]. On the whole, the electrical conductivity of the Moon is 100 times lower than inside the Earth [42].

Consideration of the magnetic properties of the Moon leads to the /30 conclusion that the outer 300 - 400 km contains about 2% metallic iron, that the material of this layer is below the Curie point and is magnetized randomly. The intensity of the local magnetic fields is evidently correlated with the abundance of breccias, which contain more of the free metal than ejected rocks. The maximum field intensity is more than 300 γ (1γ = 10^{-5} gc) at the landing site of Apollo-16, where there is very much breccia, and only 4γ at the landing site of Apollo-15, where a larger part of the surface is filled with mare basalts. Satellite magnetometers note similar differences between the visible hemisphere, where the maria are much filled with lava, and the back side covered with shattered rocks. The cause of the present magnetization of lunar rocks is still not clear [44]. It is possible that the mechanism of the self-excited dynamo in the liquid electrically-conducting iron core of the Moon existed in the past. A dipole magnetic field of the Moon has not been observed at present [12].

Thermal flux from the interior of the Moon. The thermal flux has been measured directly on the Moon at 2 points so far: in Hadley Rille at the margin of Mare Imbrium (Apollo-15) and in the mountainous

region Taurus-Littrow adjacent to Mare Serenitatis. Both regions are genetically similar in that they are at the boundaries of mascons and highland regions, where in the impact maria, with which the mascons are related, the radioactive element content is somewhat greater as compared to the average level. The temperature probes buried in the lunar soil to 1.5 - 2.5 m, where the diurnal variations of temperature are no longer felt, have shown an increase in temperature with depth. The thermal flux at both points was 28 - 30 $erg/cm^2 \cdot sec \approx 0.7 \cdot 10^{-6}$ cal/cm^2 (sic) [45, 12]. These values coincided with the magnitude of the thermal flux determined by V.S. Troitskiy and his co-workers 10 years earlier with the help of radioastronomical investigations at various wavelengths [46]. It is true that the apparatus of Troitskiy gave the average flux for the entire visible disc of the Moon. In both cases, the thermal flux obtained by independent methods significantly exceeds the flux for the chondrite model of the Moon.

Thermal history of the Moon. It is still difficult at present to select a thermal history of the Moon, which would take into account all the observed peculiarities of its structure and chemical composition, in particular its large-scale inhomogeneities. Spherically symmetric models of the Moon with a spherically symmetric temperature distribution have usually been considered up to now. The choice of the best thermal history can designate only the choice of one of such schemes for heating and cooling of the Moon, which would give the correct interval of ages for melting of the mare basalts, an age for the melting of the highland crust within the "Wasserburg gap" (cf. Figure 1.11), the correct value of the present thermal flux and the thickness of the solid shell of at least 600 - 800 km. Such a thermal history is possible only for specific values of the content of radioactive elements in the Moon and for certain limitations on the initial temperature of the Moon. An example of a thermal history, roughly satisfying these requirements, is presented in Figure 1.18, compiled from the calculations of S.V. Mayevaya [47]. The melting temperature of the lunar material is taken from the data of [48]. The step-by-step melting of first the highland crust and then the mare basalts is possible if the initial temperature of the Moon $T(t_0)$ ($t_0 = 4.5 \cdot 10^9$ years ago) has a "table" distribution (curve 1) or curve 1' decreasing toward the center, which is typical for a rapid accumulation with a

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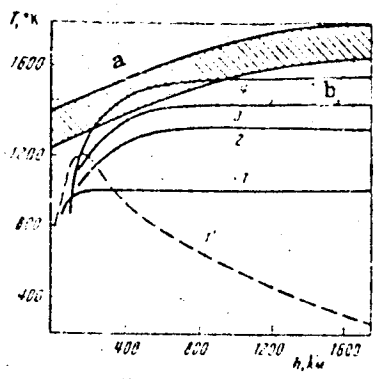

Figure 1.18. Scheme for melting of the spherically symmetric Moon with initial melting of the shell about 4.2 billion years ago: 1. 1' - different types of distribution curves for the initial temperature of the Moon; 2-4 - temperature distribution of the Moon at various times (0.2, 0.3, 0.4 billion years respectively). a- Complete melting; b- Beginning of melting.

time of $< 10^3$ years. Then the deeper layers melt later. The average radioactive element content in the Moon in this version is the following: $U = 7.7 \cdot 10^{-8}$ gm/gm, Th/U = 4, K/U = 2600. In order that the melting of the highland crust occurred no later than $(0.3 - 0.4) \cdot 10^9$ years, and the melting of the mare basalts no later than $(0.7 - 0.8) \cdot 10^9$ years from the time of the accumulation of the Moon, it is necessary that the "initial" temperature of the Moon be about 1000°K. The possibility of producing such a temperature is discussed in the 5th chapter. This scheme can be matched with the schemes of McConnel and Gast [49] (Figure 1.19) and Toksoz and Johnston [41] (Figure 1.20), unless melting of the upper layer of the Moon is assumed at the very beginning (noted in Figure 1.19 by the asterisk).

Of course, both schemes give only a model of the thermal evolution averaged over the sphere, not taking into account the latitude inhomogeneities. The problem of the true thermal and chemical evolution of the Moon requires further investigation.

It should be noted that of all versions of thermal evolution of the Moon considered in the years up to its direct conquest, the closest to the present requirements is the version of O.I. Ornatskaya and Ya.I. Al'ber, which was calculated in 1965 with consideration of the data of V.S. Troitskiy and V.D. Krotikov [46] on the thermal flux of the Moon. They obtained a time of the maximum melting of less than $1 \cdot 10^9$ years from the initial moment, a concentration $U = 7.75 \cdot 10^{-8}$ gm/gm for Th/U = 4, the present thickness of the solid crust of 600 - 700 km, the present thermal flux of $(0.8 - 0.9) \cdot 10^{-6}$ cal/cm^2·sec^{-1} [50]. /33

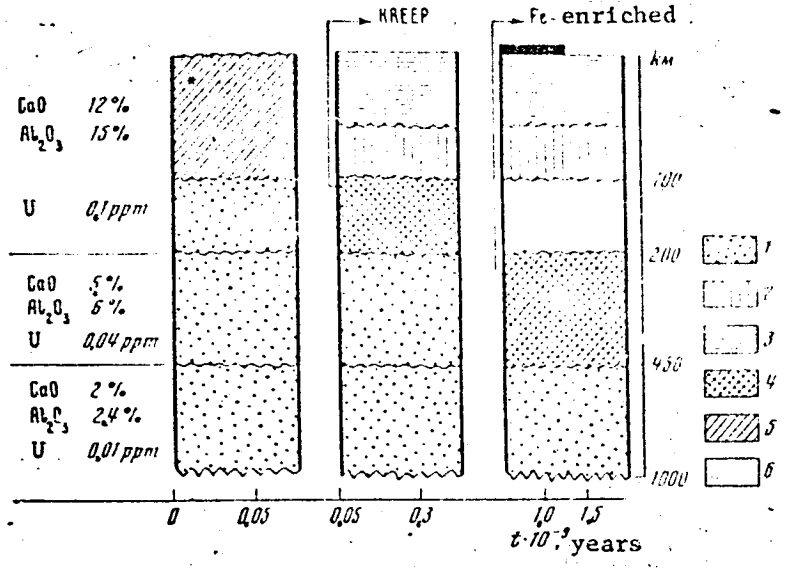

Figure 1.19. Scheme for chemical differentiation of the Moon. Substance: 1- solid; 2,3- enriched with olivine and plagioclase respectively; 4- partially molten; 5- crystallizing liquid; 6- solid residue.

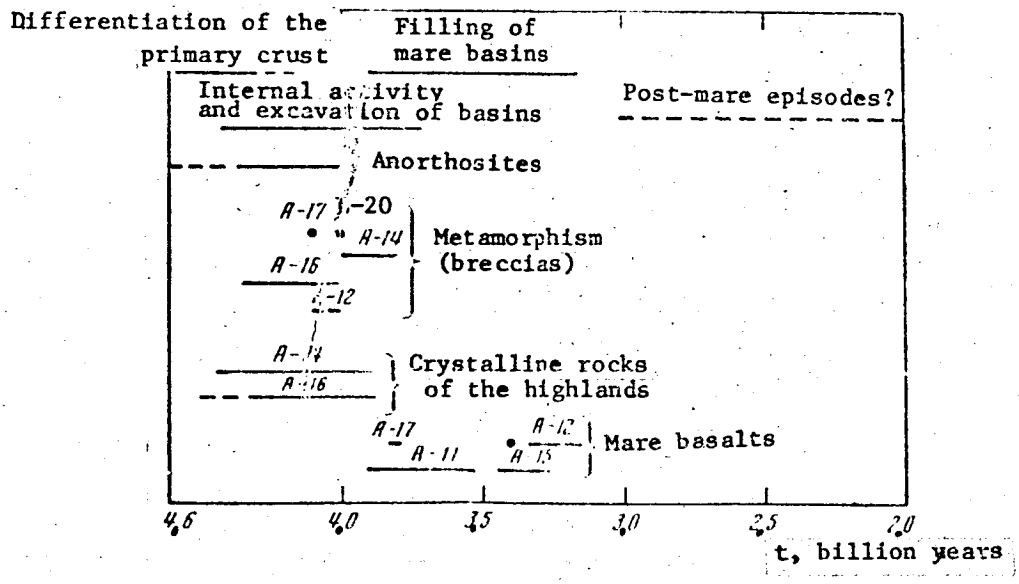

Figure 1.20. Sequence of events of the Moon.

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CHAPTER 2

TIDAL EVOLUTION OF THE EARTH-MOON SYSTEM

The study of the tidal evolution of the Earth-Moon system can /35
throw additional light onto the history of these objects. The observed lag of the ocean and solid tides in the Earth with respect to the culmination of the Moon and Sun, and also the secular deceleration of the rotation of the Earth revealed from the secular acceleration of the motions of the Sun, Moon and planets through the stars are the actual basis for constructing the theory of tidal friction and the related secular changes in the Earth-Moon system. It will probably become possible in the next decades to measure directly the rate of motion of the Moon away from the Earth with the help of laser ranging of the Moon. The theory of tides was produced by many classics of physics, beginning with I. Newton. The fundamental investigations of G. Darwin, who first considered the problem of the tidal evolution of the lunar orbit [1,2], has greatest importance for us.

Tidal evolution includes the history of the lunar orbit, the history of the rotation of the Earth and Moon around their axes, the conversion of tidal energy into heat and the role of this energy in the thermal evolution of the Earth and Moon. These problems are closely interrelated; thus, tidal theory must give agreement with the facts in all aspects.

The present motion of the Moon in its orbit about the Earth is well known with great accuracy. It is described by very complex equations, in which account is taken of the disturbances from the Sun, planets and the oblateness of the Earth. In the first approximation, the orbit of the Moon is usually defined as an ellipse with a rotating line of apsides, i.e. the major axis of the ellipse (one rotation per 8.85 years). The deviations of the real motion from the rotating ellipse are almost periodic in nature [3]. At present, the average orbit of the Moon has a semi-major axis a = 384 400 km, which is 60.3 times greater than the average radius of the Earth, and an eccentricity

e = 0.0549. The magnitude of the latter varies from 0.044 to 0.072 with a period of 8.85 years. The angle of inclination of the orbit to the terrestrial equator varies with another period. The axis of rotation of the Earth E, as is shown in Figure 2.1, is inclined to the axis of the ecliptic P by 23°27'. The inclination of the lunar orbit to the plane of the ecliptic is 5°9'. The axis of the orbit of the Moon M precesses with a period of 18.6 years about the axis of the ecliptic under the influence of the Sun. As a result, the angle between M and E or, the same thing, the angle between the planes of the lunar orbit and the terrestrial equator periodically varies between 23°27' ± 5°9'. Short-period effects of nutation are superimposed on this principal motion of the axis of the lunar orbit. Even in the course of the longest of the periods of variation of the parameters of the lunar orbit — 26,000 years, i.e., the present period of precession of the terrestrial axis — the changes contributed by tidal friction are negligibly small. Thus, the terms in the equations taking into account tidal changes are usually disregarded in theories of the motion of the Moon. On the other hand, if we want to investigate only the effect of tidal friction on the lunar orbit, which appears significantly over a time of 10^8 - 10^9 years, then we can neglect all the periodic variations and consider some average orbit of the Moon for each geological epoch. We will first consider the Moon as the satellite of the Earth for the most remote epochs, when it was closer to the Earth. This follows from the celestial mechanical analysis of the motion of the Moon: even at the present distance from the Earth the Moon is its stable (according to Hill) satellite, which can remain for an indefinite time within some finite band circling the Earth with its motion around the Sun [3].

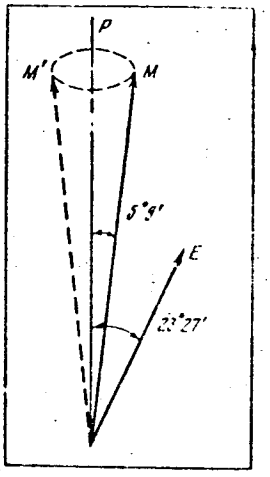

Figure 2.1. Relative position of the axes of rotation of the Earth E, lunar orbit M and ecliptic P.

We will consider here the tidal evolution of the Earth-Moon system in three stages: first - the simplest case of a circular lunar orbit in the equatorial plane of the Earth, considering the Moon as a material point, and the Earth-Moon system as isolated in space.

This gives a clearer representation about the rate of change in the distance of the Moon from the Earth depending on the dissipative properties of the Earth, and also permits evaluating the contribution of tidal friction in the heating of the terrestrial interior [4]. The next stage will be a consideration of the motion of the Moon in an elliptical orbit in the equatorial plane of the Earth, also for the case of a two-body system.

Information about the tidal changes of the eccentricity of the lunar orbit with time permits judging the possibility of the approximate representation of the lunar orbit as a circle.

By considering the Moon as a deformable body, one can evaluate /37
the change with time of the eccentricity of the lunar orbit, by relating it to the dissipation of tidal energy within the Moon. The fact that for the most probable models of the internal structure of the Moon the eccentricity of its orbit was even less in the past than at present, permits considering it circular to a good approximation [5,7]. And this in turn makes it possible to investigate rather completely the evolution of the angle of inclination of the lunar orbit with respect to the terrestrial equator and to the ecliptic with consideration of the forces of the Sun [8]. The orientation of the lunar orbit with respect to these two principal planes is a very important characteristic. It permits limiting the region of possible initial distances of the Moon from the Earth and thereby clarifies the mechanism of the formation of the Moon.

We will also discuss the relative role of the ocean and solid tides in the Earth and the possible role of tidal friction in creating and maintaining the temperature inhomogeneities in the interior of the Earth.

2.1. Tidal Evolution of a Circular Satellite Orbit in the Equatorial Plane of the Planet.

Let us consider the simplest case of the circular orbit of a point satellite with mass m moving about a slightly oblate rotating planet M in its equatorial plane at a distance r from the center of

the planet. We denote the angular velocity of rotation of the planet by Ω and the angular velocity of revolution of the satellite in the orbit by $\omega; \omega \neq \Omega$. The planet will be assumed a deformable body, whose properties differ somewhat from an ideal elastic body, which is characterized by a Q-factor $Q < \infty$. The quantity Q is related by the following approximate relation to the angle δ of lag (for $\Omega > \omega$) or lead (for $\Omega < \omega$) of the solid tide with respect to the culmination of the tide-forming body:

$$Q \approx \frac{1}{\operatorname{tg} 2\delta} \approx \frac{1}{2\delta}, \tag{2.1}$$

if this angle is small (Figure 2.2)*. The quantity 1/Q is an integral measure of the dissipation of tidal energy in the interior of the planet [9]. The tidal energy is dissipated in the Earth principally due to the ocean tides in low-water channels and basins. However, the total effect of the ocean and solid tides can be characterized by some effective lag angle, which the solid Earth without oceans would have for the equivalent tidal interaction with the Moon.

The planet-satellite system will be assumed isolated in space. /38
The expression for the tide-forming force at the surface of the planet at a distance R from its center can be obtained from the equation for the tide-forming potential [10]:

$$V = \frac{GmR^2}{r^3}\left[P_2(0) + \frac{R}{r}P_3(0) + \ldots\right] \quad \sum_{n=2}^{\infty} V_n,$$
$$V_n = \frac{GmR^n}{r^{n+1}} P_n(0), \tag{2.2}$$

where V_n is the tide of n-th order; P_n is the Legendre polynominal of order n; θ is the angle between the direction to the point under consideration and the direction toward the satellite from the center of the planet; r is the distance of the satellite from the planet; G is the gravitational constant. In practice, only the first term is often retained in expression (2.2):

$$V \cong V_2 = \frac{GmR^2}{r^3}\frac{1}{2}(3\cos^2\theta - 1). \tag{2.3}$$

The term V_3 is about 2% of V_2 in the present Earth-Moon system; in past epochs, for closer distances of the Moon to the Earth, its relative and absolute values were larger. V_3 was taken into account in

* In the case of lag the tidal bulge is ahead, and in the case of lead is behind, with respect to the line joining the centers of the planet and satellite.

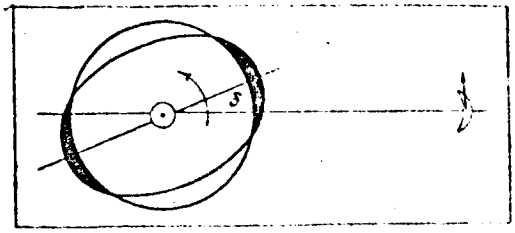

Figure 2.2. Diagram of the deflection of the tidal bulges from the direction toward the tide-forming object.

the calculation of Mac Donald [11]. In this paragraph, giving the simplest representation of the tidal evolution, we will use only the term V_2.

Since the present flattening of the Earth is small - about 1/300, then the shape of the Earth, not subjected to the tidal effect, can be taken as a sphere with sufficient accuracy (to 0.3%) in the theory of the terrestrial tides, while the changes in shape caused by the tides and calculated for a sphere can be considered superimposed on the effective spherical shape.

Because of the tidal deformation of the Earth, its attraction changes, and additional forces arise, which are characterized by an additional potential V'. According to Love, an additional potential V'_n from the corresponding deformations of the Earth arises from each spherical function V_n in the tidal potential for the elastic Earth, where:

$$V'_n = k_n V_n, \tag{2.4}$$

where k_n is the Love function depending on the elastic models and distribution of mass within the Earth. /39

If only the first term of the expansion is again retained, then the additional potential at the spherical surface of the Earth will be:

$$V'_2 = \frac{k_2 G m_1 R^2}{r^3} P_2(0), \tag{2.5}$$

and in the space beyond at a distance r from the center of the Earth:

$$V'_2(r, \theta) = \frac{k_2 G m_1 R^2}{r^3} \left(\frac{R}{r}\right)^3 P_2(\theta). \tag{2.6}$$

Thus, the additional potential at the distance of the Moon in the first approximation equals:

$$V'_2(r, \theta) = \frac{k_2 G m_1 R^2}{r^3} \frac{1}{2} (3\cos^2\theta - 1). \tag{2.7}$$

The lag of the tides, i.e., the deviation of the maxima of the tidal deformations from the line of centers of the Earth and Moon, produces a force couple, on whose basis is obtained the torque L:

$$\mathbf{L} = mr\mathbf{F}_\theta, \tag{2.8}$$

whose absolute value has the form:

$$L = \frac{rm}{r}\frac{dV_2}{d\theta}\Big|_{\theta=\delta} = \frac{3}{2}\frac{k_2Gm^2R^5}{r^6}\sin 2\delta. \tag{2.9}$$

This torque causes a redistribution of the angular momentum in the two-body Earth-Moon system, while the total angular momentum remains constant:

$$H + h = I\Omega + \frac{Mm}{M+m}r^2\omega = \text{const} = K, \tag{2.10}$$

$$-\frac{dH}{dt} = +\frac{dh}{dt} = L. \tag{2.11}$$

Here H is the rotational angular momentum of the Earth, h is the orbital angular momentum of the Moon, I is the moment of inertia of the Earth. The vectors H and h are collinear in this system; thus, we will make use of their scaler values. The rotational angular momentum of the Moon is several orders of magnitude smaller than H and h, and will thus be neglected. One should also join to this system of equations Kepler's third law:

$$\omega^2r^3 = G(M+m) = \mu. \tag{2.12}$$

With the angular velocity of rotation of the Earth Ω greater by 5 - 6 times in the past, I must have exceeded its present value by 6 - 6% [5, 12]. The moment of inertia of the Earth could also have decreased somewhat with time due to gravitational differentiation of the terrestrial interior. We will take for simplicity that the value of I does not depend on time. Then equations (2.9) and (2.11) give us an expression for the secular decrease of the angular velocity of rotation of the Earth [10]: /40

$$\frac{d\Omega}{dt} = -\frac{3}{2}\frac{k_2Gm^2R^5}{Ir^6}\sin 2\delta, \tag{2.13}$$

and we obtain from (2.10), (2.12) and (2.13) the related increase of the radius of the lunar orbit:

$$\frac{GMm}{2I\mu^{1/2}r^{1/2}}\frac{dr}{dt} = -\frac{d\Omega}{dt}, \quad \frac{dr}{dt} = \frac{3km^2R^5\sin 2\delta\mu^{1/2}}{Mr^{11/2}}. \tag{2.14}$$

The dependence of the tidal withdrawal of the Moon on the lag angle can be written in the form:

$$\int_{r_0}^{r} r^{11/2}\,dr = A\int_{t_0}^{t}\sin 2\delta\,dt, \tag{2.15}$$

where

$$A = 3k_2mR^5\mu^{1/2}/M.$$

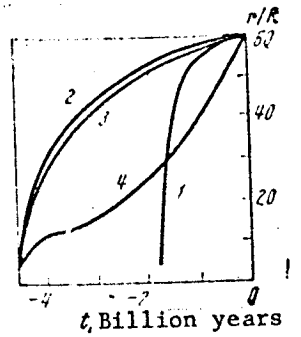

Figure 2.3. Dependence of the change in the Moon-Earth distance with time on the effective lag angle:

1- δ = const = 2°,16; 2- $\delta = 1°,7\ (1 - t/t_{init_2})$; 3- $\delta = 0°,5 + \beta(t-t_{init})$; $\delta_{pres} = 2°,16$; 4- $\delta = 0",02 \exp \gamma(t - t_{init})$; $\delta_{pres} = 10°$ from newer data [13], $\delta_{pres} = 4°$.

In spite of the simplified nature of the derivation, the integration of equation (2.15) leads to consequences important for understanding the essence of the tidal evolution in the real case. Firstly, the duration of the tidal evolution from the closest distance to the Earth to its present orbit is much less than the age of the Earth-Moon system, if one assumes the angle δ (or equivalently, the dissipation function 1/Q) for the whole Earth as independent of time and equal to its present value. According to the data of Mac Donald, the present value $\delta_{pres} = 2°, 16$ [15]; δ = const, we obtain:

$$r^{11/2} = r_0^{11/2} + \frac{13}{2} A \sin 2\delta (t - t_0). \qquad (2.16)$$

By setting $r_0 = 0$ at $t_0 = 0$ (which gives the maximum duration of the withdrawal), we obtain $t = 1.75 \cdot 10^9$ years for $\delta = 2°, 16$ and $r = 60.3$ R, which is significantly less than the age of the Earth and Moon, which is $4.5 - 4.6 \cdot 10^9$ years (Figure 2.3, curve 1). The new values $\delta \approx 4°$ would lead to an even shorter tidal scale of about $0.9 \cdot 10^9$ years.

The duration of the tidal evolution can be matched with the age of the system only under the assumption that the effective lag angle of the tides was less in the past than at present. Such an assumption agrees qualitatively with present ideas about the thermal history of the Earth, whose interior gradually heated from a lower temperature, while releasing the water of the oceans to the surface. Then δ must increase with time due to the growth of the volume of the oceans. However, it is still difficult to determine the dependence $\delta(t)$ in a quantitative form. One can only select simple dependences of the growth of $\delta(t)$. Some of these (linear, quadratic and exponential growth laws of δ with time) have given a duration of the tidal evolution of the Earth-Moon system equaling 4.5 billion years for present /41

values of δ of 1.7, 2.16 and 10°, respectively [4] (Figure 2.3, curves 2 - 4).

Secondly, it is seen from Formula (2.15) and Figure 2.3 that the duration of the tidal withdrawal very strongly increases with increasing r. For a constant δ, the transition time Δt from an orbit of radius r_1 to an orbit of radius r_2 is proportional to $r_2^{11/2} - r_1^{11/2}$. For δ increasing with time, this dependence becomes less sharp, but nonetheless the duration of the tidal withdrawal of the Moon within the first 20 Earth radii remains much less than the age of the system [if one does not consider the case, which is not very realistic from the geophysical point of view, when δ increases with time according to an exponential law and reaches a present value of 10° (Figure 2.3, curve 4)].

Thus, the tidal evolution of the Earth-Moon system must have proceeded very nonuniformly. The period of withdrawal of the Moon, from some unknown initial distance close to the Earth to approximately half the present distance (i.e., 30 R), can be called the time of rapid changes, in contrast to the following period, which has lasted almost the entire lifetime of the Moon. The nonuniformity of the tidal withdrawal greatly hinders the determination of the initial distance of the Moon from the Earth. One can try to find this important characteristic by other methods, which will be discussed below.

Thirdly, a monotonic dependence r(t) is always obtained for a constant sign of $\delta(t)$. The constant sign of $\delta(t)$ is ensured by the constant sign of the difference $(\Omega - \omega)$, which always remains positive during the tidal withdrawal of the Moon from the Roche limit (2.80 — 2.89)R to its present distance [cf. (2.20)]. Thus, it will be convenient to relate all the orbital parameters of the Moon not to time, but to its distance from the Earth or to the semimajor axis of the orbit. This is possible, since within the framework of the theory of weak friction, i.e., a small value of the lag angle of the tidal bulge with /42
respect to the culmination of the tide-forming body, the relation between the semimajor axis, eccentricity and inclination of the orbit does not depend on the duration of the evolution. The case of a circular equatorial orbit of the Moon permits simple evaluation of the energy contribution of tidal friction in the thermal balance of the

Earth. The heat evolved with the dissipation of tidal energy is derived from the kinetic energy of rotation of the Earth, which slows down.

The change in the energy of rotation of the Earth equals:

$$\frac{dE_{rot}}{dt} \quad \frac{d}{dt}\left(\frac{I\Omega^2}{2}\right) = I\Omega\frac{d\Omega}{dt} < 0. \tag{2.17}$$

(Here we have again neglected the small relative change of I, which underestimates the value of dE/dt by only several percent in the remote past.) Part of this amount is expended in the increase of the total orbital energy of the Moon E_{orb} with its withdrawal from Earth; the remaining part must be dissipated in the interior of the Earth:

$$E_{orb} = E_{kin} + E_{pot} = GmM/2r - GmM/r = -GmM/2r,$$
$$\frac{dE_{orb}}{dt} = \frac{GMm}{2r^2}\frac{dr}{dt}. \tag{2.18}$$

By making use of the Relations (2.17), (2.12) — (2.14), we obtain an expression for the energy dissipated per unit time [4]:

$$\left(\frac{dE}{dt}\right)_{therm} = -\left(\frac{dE}{dt}\right)_{rot} - \left(\frac{dE}{dt}\right)_{orb} = -I\Omega\frac{d\Omega}{dt} - \frac{GMm}{2r^2}\frac{dr}{dt} =$$
$$= +\frac{3}{2}\frac{kGm^2R^5\sin 2\delta}{r^6}(\Omega - \omega). \tag{2.19}$$

It is seen from this expression that the energy release within the planet is proportional to the first power of the lag angle δ (since for small angles $2\delta \approx 2\delta$) and is inversely proportional to the sixth power of the distance of the satellite. It is divided between the oceans and the solid Earth proportionally to their contribution in δ_{eff}. The value of $(\Omega - \omega)$ also increases almost consistently with decreasing distance (Figure 2.4), which makes the stage of the close position of the Moon, when the release of tidal energy was maximum, most interesting from the geophysical point of view. Figure 2.5 shows three versions of the curves of the tidal release of heat $(dE/dt)_{therm}$ depending on time, which correspond to the linear, quadratic and exponential laws for the change of the angle δ with time (curves 2 - 4 in Figure 2.3). The figure also shows for comparison the release of energy by radioactive elements for two versions of their content in the Earth, close to the chondrite content (1 — $U = 10^{-8}$ gm/gm, 1' — $U = 2 \cdot 10^{-8}$ gm/gm).

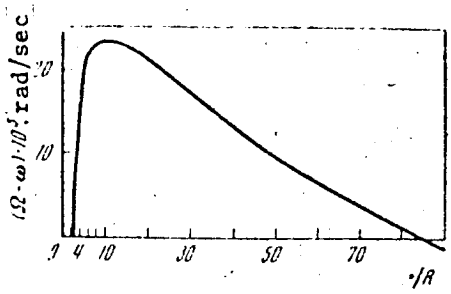

Figure 2.4. Dependence of the difference in angular velocities of rotation of the Earth and revolution of the Moon (Ω—ω) on the distance from the Earth expressed in Earth radii (circular orbit in the plane of the equator).

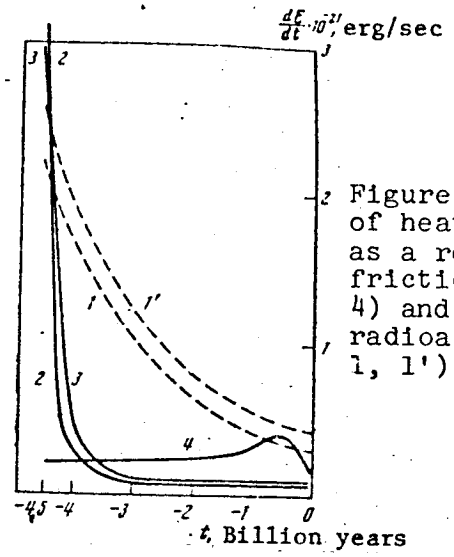

Figure 2.5. Release of heat in the Earth as a result of tidal friction (curves 2 - 4) and because of radioactivity (curves 1, 1').

Let us now evaluate the integral contribution of tidal energy to the thermal balance of the Earth.

We express $(\Omega - \omega)$ with the help of (2.10) and (2.12) in the form: /43

$$(\Omega - \omega) = [K - I\mu^{1/2}r^{-3/2} - Mm(M+m)^{-1}r^{1/2}\mu^{1/2}]I^{-1}. \qquad (2.20)$$

One can find from equation (2.20) the condition for the difference $(\Omega - \omega)$ to vanish, which corresponds to the revolution of the satellite being synchronized to the rotation of the planet: the satellite would "hang" above one point of the equator. A synchronous orbit has never been achieved in the actual Earth-Moon system; instead, there are the so-called distances of "closest approach" and "maximum withdrawal" (cf. Chapter 3). But if one neglects the inclination and ellipticity of the lunar orbit and substitutes into equation (2.20) for the idealized system the numerical characteristics of the real Earth-Moon system (M, m, K, I), then one can find the two values of r: r_{min} and r_{max} for which $(\Omega - \omega) = 0$ (cf. Figure 2.4). The close synchronous orbit is at a distance of $\sim$2.4R, i.e. within the Roche limit*, equaling 2.89 R. The far synchronous orbit is at a distance of $\sim$ 85 R, which is significantly farther than the present distance of the Moon

* The Roche limit is the distance of a satellite from a planet, at which its tidal forces tending to break up the satellite equal the gravitational forces holding the satellite together. For a liquid satellite with the mass and density of the Moon, the Roche limit is 2.89 R.

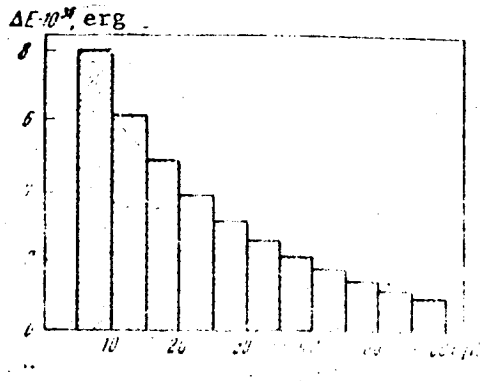

Figure 2.6. Release of tidal heat in the Earth for different stages of withdrawal of the Moon.

of 60.3 R. In the interval between these values $(\Omega - \omega) > 0$, which also defines the monotonic course of dr/dt with time.

In the isolated two-body system /44
the state for $(\Omega - \omega) = 0$ is stable; tidal withdrawal is absent. For $(\Omega - \omega) < 0$, tidal approach occurs, which is accompanied by an increase of Ω and ω, so that the difference preserves the sign. Thus, a transition from one type of tidal evolution to the other is impossible. Tidal approach is also observed for Ω and ω having opposite directions (counter-rotation). In the solar system, the state $(\Omega - \omega) < 0$ exists only for the system of Mars and Phobos, and counter-rotation with a large satellite only in the system of Neptune and Triton. Tidal approach occurs in both cases.

By using the fact that the function r(t) is monotonic in the interval from r_{min} to r_{max}, we can express the release of tidal energy in a planet depending on the increase of r:

$$\frac{d\,E_{therm}}{dr} = \frac{d\,E_{therm}}{dt} : \frac{dr}{dt}$$

where dr/dt is given by Formula (2.14). With consideration of (2.20), we obtain the integral release of heat in the planet with withdrawal of the satellite from a distance r_0 to r, which is convenient in that the unknown past value of δ does not appear in it:

$$E_{therm} = \frac{1}{2}\int_{r_0}^{r}\left(\frac{GMmK}{\mu^2 + r^2} - \frac{GMm}{r^2} - \frac{GM^2m^2}{(M+m)r}\right)dr. \qquad (2.21)$$

Relation (2.21) can be applied as a sufficient approximation to the real Earth-Moon system, since consideration of the non-collinearity of the angular momentum vectors H and h introduces an error no greater than 1 - 2% in evaluating E_{therm} (the ellipticity of the orbit of the Moon affects E_{therm} even less). Figure 2.6 presents a diagram of the release of tidal energy in the Earth depending on the interval of distances over which the tidal withdrawal occurs (taken in intervals of

5 R). The integral release of heat for r_0 = 10 R and r_{pres} = 60.3 R is $E_{therm} = 2.8 \cdot 10^{37}$ ergs. By taking the Roche limit of 2.89 R as r_0, we would obtain $E_{therm} = 4.0 \cdot 10^{37}$ ergs. The release of heat by radioactive sources over the entire history of the Earth is estimated as $(1.2 - 1.5) \cdot 10^{38}$ ergs. By taking into account that the larger part of the tidal energy is dissipated in the oceans and is rapidly radiated into space, one can conclude that the tidal heat has made and makes only a small contribution to the total internal heat of the Earth, if one excludes the short-term stage of the very close orbit of the Moon. /45

At the same time, if the tidal heat is released in the Earth not uniformly over the volume, but concentrated in some layer, it can play a significant role in the production and maintenance of temperature inhomogeneities of the Earth (cf. 2.5 of this chapter).

2.2. Tidal Evolution of an Elliptical Satellite Orbit in the Equatorial Plane of the Planet.

The present eccentricity of the lunar orbit is small (e_{pres} = 0.0549) and in calculations of tidal evolution the lunar orbit can be considered approximately circular. However, such an approximation requires proof for past epochs.

As in the case of the circular orbit, the principle role in the secular changes in the ellipticity of the orbit of the satellite is played by the lag of the diurnal and semidiurnal tides on the planet, which produces the torque L and converts part of the rotational angular momentum of the planet H into orbital angular momentum h. The difference from the case of the circular orbit is that for an elliptical orbit:

$$h = Mm\sqrt{\frac{Ga(1-e^2)}{(M+m)}}, \quad E = -\frac{GmM}{2a}, \tag{2.22}$$

where a is the semimajor axis. An increase of h with a simultaneous increase of the orbital energy E is accompanied by an increase of a. The eccentricity of the orbit e must increase at the same time. This occurs because the tidal interaction increases inversely proportionally to the sixth power of the distance between the objects, and even for

a small ellipticity of the orbit the satellite at the pericenter must be pulled forward significantly more strongly than at the apocenter. With the general elongation of the orbit, its similarity is not preserved: each successive apocenter is further from the preceding pericenter. This also leads to an increase of e with time. The proof of this statement with the help of equations is given in works [5,6,14].

With the motion of a satellite along an elliptical orbit, there is also an opposite effect, which decreases e and a with time. Because of the periodic change in distance between the satellite and the planet, the tidal bulges on both bodies experience radial pulsations with a monthly period. The height of the bulges is inversely proportional to the cube of the distance between them, which makes the effect significant even for a small eccentricity of the orbit. Because of inelasticity, the pulsation maxima must begin after the passage of the satellite through the pericenter, and the minima — after apocenter. In view of the purely radial nature of these pulsations, their lag does not cause the torque, and consequently, neither the orbital nor the rotational angular momenta h and H change, but the energy dissipated in the interior of both bodies must be derived from the orbital energy of the satellite. According to (2.22): /46

$$\frac{dE}{dt} = + \frac{GmM}{2a^2} \frac{da}{dt}. \tag{2.23}$$

This effect appears in pure form for h = const (both bodies turn one face toward the other). Then dE/dt < 0, and consequently, one must also have da/dt < 0, i.e., a decrease in time of the semimajor axis a because of the dissipation of the energy of the radial tides. It is easily seen from Formula (2.22) for the orbital angular momentum h that for h = const and da/dt <0 there must also occur a decrease in the eccentricity of the orbit e.

Let us turn to the total estimate of the tidal changes of a and e for elliptical motion with h ≠ const. The Lagrangian equations for the osculatory elements in perturbed plane motion have the form [15]:

$$\begin{aligned} \frac{da}{dt} &= \frac{2a^2 e \sin v}{p} \bar{S} + \frac{2a^2}{r} \bar{T}, \\ \frac{de}{dt} &= \bar{S} \sin v + \left[\cos v + (e + \cos v) \frac{r}{p}\right] \bar{T}. \end{aligned} \tag{2.24}$$

Here:

$$p = a(1 - e^2), \quad r = a(1 - e^2)/(1 + e\cos v),$$

v - the true anomaly, i.e., the angle formed by the radius vector of the satellite and the direction of the origin for the motion along the orbit:

$$\widetilde{S} = \frac{\sqrt{p}}{\sqrt{\mu}} S, \quad \widetilde{T} = \frac{\sqrt{p}}{\sqrt{\mu}} T, \quad \mu = G(M + m).$$

S is the radial and T is the transverse components of the perturbing forces. The component T can be found by differentiating Expression (2.8) for the tidal potential with respect to the angular coordinate:

$$T = \frac{1}{r}\frac{dV}{d\psi}\Big|_{\psi=\delta} = -\frac{3}{2}\frac{GmR^5}{r^4}\sin 2\delta. \tag{2.25}$$

This expression is applicable both for the case of a planet with a satellite in the form of a material point and for a planet with a deformable satellite in synchronized rotation, such as the Moon.

The radial perturbing force acting on the satellite is composed of the two components caused by the tidal deformation of the planet and satellite. Each of them is obtained by differentiating the corresponding potentials with respect to the coordinate along the line joining the centers of the bodies: /47

$$S = \frac{\partial V_\oplus}{\partial r} + \frac{\partial V_\zeta}{\partial r}\frac{M}{m} =$$
$$= -\frac{3k_\oplus GmR_\oplus^5}{r^4}(3\cos^2\delta_\oplus - 1) - \frac{3k_\zeta GMR_\zeta^5}{r^4}\frac{M}{m}(3\cos^2\delta - 1), \tag{2.26}$$

where dV_ζ/dr is the force acting per unit mass on the Earth. The force acting per unit mass on the Moon is obtained by multiplying dV_ζ/dr by M/m.

After substituting Expressions (2.25) and (2.26) for the perturbing forces S and T into Equations (2.24) we obtain:

$$\frac{da}{dt} = \alpha_{11}a^2\frac{\sqrt{p}}{r^2} - (\alpha_{12} + \alpha_{13})\frac{a^2}{\sqrt{p}}\frac{e\sin v}{r^2},$$
$$\frac{de}{dt} = \alpha_{21}\sqrt{p}\left[\cos v + (e + \cos v)\frac{r}{p}\right]\frac{1}{r^2} - (\alpha_{22} + \alpha_{23})\frac{\sqrt{p}}{r^2}\sin v. \tag{2.27}$$

Here:

$$\alpha_{11} = 3k_\odot GmR_\odot^5\sin 2\delta_\odot/\sqrt{\mu},$$
$$\alpha_{12} = 6k_\odot GmR_\odot^5(3\cos^2\delta_\odot - 1)/\sqrt{\mu},$$
$$\alpha_{13} = 6k_\zeta GM^2R_\zeta^5\cdot(3\cos^2\delta_\zeta - 1)/m\sqrt{\mu},$$
$$\alpha_{21} = {}^3/_2 k_\odot GmR_\odot^5\sin 2\delta_\odot\sqrt{\mu}, \tag{2.28}$$
$$\alpha_{22} = 3k_\odot GmR_\odot^5(3\cos^2\delta_\odot - 1)/\sqrt{\mu},$$
$$\alpha_{23} = 3k_\zeta GM^2R_\zeta^5(3\cos^2\delta_\zeta - 1)/m\sqrt{\mu}.$$

The effect of the tidal friction develops extremely slowly. Thus, it is suitable to average Equation (2.27) by assuming that over the time τ of one revolution of the satellite a and e remain constant. Then:

$$\left[\frac{da}{dt}\right]_\tau = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{da}{dt}\right)d\theta,$$

$$\left[\frac{de}{dt}\right]_\tau = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{de}{dt}\right)d\theta, \tag{2.29}$$

where θ is the mean anamoly related to the average angular velocity of the satellite over the orbit n by the expression:

$$nd\theta = dv. \tag{2.30}$$

By taking into account that:

$$\frac{dv}{dt} = \frac{\sqrt{\mu p}}{r^2},$$

we obtain:

$$\left[\frac{da}{dt}\right]_\tau = \frac{1}{2\pi}\int_0^{2\pi}\frac{r^2}{a^2\sqrt{(1-e^2)}}\left(\frac{da}{dt}\right)dv,$$

$$\left[\frac{de}{dt}\right]_\tau = \frac{1}{2\pi}\int_0^{2\pi}\frac{r^2}{a^2\sqrt{(1-e^2)}}\left(\frac{de}{dt}\right)dv. \tag{2.31}$$

Substituting the expressions for da/dt and de/dt from (2.27), we have:

$$\left[\frac{da}{dt}\right]_\tau = \frac{1}{2\pi}\int_0^{2\pi}\frac{r^2}{a^2\sqrt{1-e^2}}\left[\varkappa_{11}a^2\frac{\sqrt{p}}{p^2}(1+e\cos v)^6 - \right.$$
$$\left. -(\varkappa_{12}+\varkappa_{13})\frac{a^2e}{\sqrt{p}}\frac{(1+\cos v)^7}{p^7}\sin v\right]dv, \tag{2.32}$$

$$\left[\frac{de}{dt}\right]_\tau = \frac{1}{2\pi}\int_0^{2\pi}\frac{r^2}{a^2\sqrt{(1-e^2)}}\left[\varkappa_{21}\sqrt{p}\left(\cos v+\frac{e\cos v}{1+e\cos v}\right)\frac{(1+\cos v)^7}{p^7} - \right.$$
$$\left. -(\varkappa_{22}+\varkappa_{23})\frac{\sqrt{p}}{p^7}(1+\cos v)^7\sin v\right]dv.$$

The terms containing sin v give zero with integration from 0 to 2π. After carrying this out [cf. (2.24)], we calculate the following integrals:

$$\mathcal{J}_1 = \int_0^{2\pi}\left(\cos v+\frac{e+\cos v}{1+e\cos v}\right)(1+e\cos v)^5dv = \pi e(11+16{,}5e^2+1{,}375e^4),$$

$$\mathcal{J}_2 = \int_0^{2\pi}(1+e\cos v)^6dv = \pi(2+15e^2+11{,}25e^4+0{,}625e^6), \tag{2.33}$$

$$\mathcal{J}_3 = \int_0^{2\pi}\frac{dv}{(1+\cos v)^2} = \frac{2\pi}{(1-e^2)^3}.$$

By substituting γ_1, γ_2, γ_3 into (2.31) we obtain:

$$\frac{\Delta a}{\tau} = \frac{\alpha_{11}}{p^6}(2 + 15e^2 + 11{,}25e^4 + 0{,}625e^6) \quad (\Delta a)_\tau$$

$$\frac{\Delta e}{\tau} = \frac{\alpha_{12}e}{p^5}(11 + 16{,}5e^2 + 1{,}375e^4) \quad (\Delta e)_\tau \qquad (2.34)$$

where $\tau = 2\pi a^{3/2}/\sqrt{\mu}$ is the period of revolution of the Moon.

We have obtained the "secular" changes of $(\Delta a)_\tau$ u $(\Delta e)_\tau$ caused by the effect of the tangential force $\tilde{T}$ only; the effect of the radial force $\tilde{S}$ is zero on the average. The effect of the lag of the maxima of the radial pulsations, i.e., the tides with a monthly period, with respect to the passage through perigee and apogee can be taken into account with the help of Equations (2.22): /49

$$(\Delta a)_m = \frac{2a^2}{GmM}(\Delta E)_m,$$

$$(\Delta e)_m = \frac{Gmp}{mM^2G^2e}(\Delta E)_m, \qquad (2.35)$$

where $(\Delta E)_m$ is the heat release in the Moon and Earth due to dissipation of the energy of the monthly tides, which do not change the value of the orbital angular momentum h.

We take from works [16, 17] the expression for the dependence of ΔE on a and e for constant dissipative properties of the Earth and Moon ($\delta_\odot = \text{const}$, $\delta_\zeta = \text{const}$):

$$\Delta E = \Delta E_0 \left(\frac{a_0}{a}\right)^{15/2} \left(\frac{e}{e_0}\right)^2, \qquad (2.36)$$

where the values with index zero refer to their present values. By denoting:

$$\alpha'_{11} = \frac{2\Delta E_0}{GmMe_0^2}, \quad \alpha'_{22} = \frac{\Delta E_0}{GmMe_0^2} \qquad (2.37)$$

and combining the values $(\Delta a)_\tau$ and $(\Delta e)_\tau$ respectively with $(\Delta a)_m$ and $(\Delta e)_m$, we obtain:

$$\Delta a = \alpha'_{11}\frac{a^2}{p^6}(2 + 15e^2 + 11{,}25e^4 + 0{,}625e^6) - \alpha'_{12}a^{3/2}e^2\left(\frac{a_0}{a}\right)^{15/2},$$

$$\Delta e = \alpha'_{21}\frac{e}{p^5}(11 + 16{,}5e^2 + 1{,}375e^4) - \alpha'_{22}pea^{3/2}\left(\frac{a_0}{a}\right)^{15/2}, \qquad (2.38)$$

where:

$$\alpha'_{11} = 2\pi\alpha_{11}\mu, \quad \alpha'_{21} = 2\pi\alpha_{22}/\mu.$$

Here the increments Δa and Δe take into account both the effect of the lag of the diurnal tides (tangential force) as well as the lag

of the radial pulsations. These increments refer to time intervals on the long (tidal) scale. On the basis of these relations, G.G. Petrova [6] has evaluated the change in the eccentricity of the orbit of the Moon in the past for the following variations (Figure 2.27):

1) $\Delta E_m = 0.1\,\Delta E_0 = 0.6 \cdot 10^{15}$ erg/sec
2) $\Delta E_m = \Delta E_0 = 0.6 \cdot 10^{16}$ erg/sec
3) $\Delta E_m = 0.8 \cdot 10^{16}$ erg/sec
4) $\Delta E_m = 10\,\Delta E_0 = 0.6 \cdot 10^{17}$ erg/sec

The value of $\Delta E_0 = 0.6 \cdot 10^{16}$ ergs/sec, according to [18], corresponds to the model of the Moon with the factor $1/Q = 0.01$ and Love number $k_{2\zeta} = 0.2$. The versions 1 and 4 correspond to a decrease by 10 times and an increase by 10 times of the value of ΔE_0. Version 3 is selected as the case giving an eccentricity approximately constant in time with $\delta_3 = \text{const} = 1'$. For the present $\delta_3 = 4'$, the analogous "equilibrium" of the effect of the tangential and radial forces would occur for ΔE_0 greater by 4 times, i.e., $0.3 \cdot 10^{17}$ erg. It is seen from Figure 2.7 that the behavior of the eccentricity of the lunar orbit in the past, i.e., for small a, depended strongly on the value of ΔE_m, which is significantly smaller (by 2 or 3 orders of magnitude) than ΔE_{tang}. However, the effect of the radial forces on the eccentricity of the orbit is comparable to the effect of the transverse forces. For large ΔE_m, i.e., dissipation of energy in the radial tides to which the tides on the Moon contribute, the eccentricity of the orbit in the past could have been significantly greater than the present (version 4). The most probable of all is the course of the eccentricity with a decrease into the past, as in versions 1 and 2 in Figure 2.7. The decrease in the eccentricity into the past was also obtained by Mac Donald [5] for a solid model of the Moon in the present and in the past (Figure 2.8). The inclination of the lunar orbit to the terrestrial equator does not change the nature of the conclusion about the evolution of the eccentricity. The maximum heating of the interior of the Moon is produced for large values of ΔE_m both at present and in the past, which could have occurred only for a molten Moon, besides moving in a very elongated orbit. However, even version 4, which obviously gives enhanced values of the dissipation of energy in the Moon and a too large change in the eccentricity, leads to an integral heating of the interior of the Moon of at most several tens

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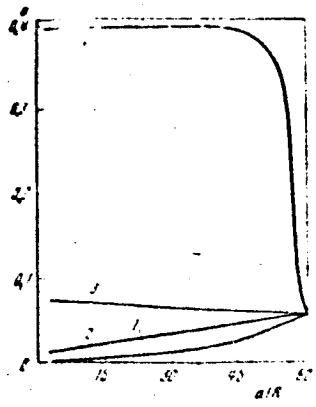

Figure 2.7. Dependence of the eccentricity of the lunar orbit on the semimajor axis a for $\delta_\oplus = \text{const} = 1°$ and different values of ΔE_m: 1 — $\Delta E_m = 0{,}6 \cdot 10^{16}$; 2 — $0{,}6 \cdot 10^{15}$; 3 — $0{,}8 \cdot 10^{16}$; 4 — $0{,}6 \cdot 10^{17}$ erg/sec.

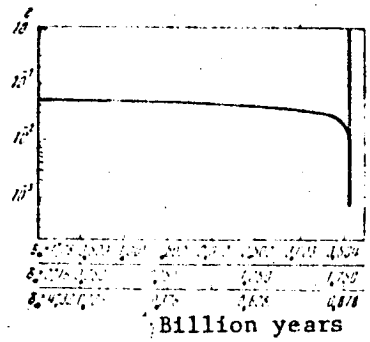

Figure 2.8. Dependence of the eccentricity of the lunar orbit e on the time in the past t for various values of δ_ϵ.

of degrees; versions 1 - 3 give even less. Thus, tidal heating leads only to an insignificant contribution to the heat of the interior of the Moon as compared to radioactivity, which could heat the Moon to temperatures of 1500 - 2000°K.

The small value of the eccentricity of the lunar orbit at present and in the past makes it possible to consider the orbit as approximately a circle. This is particularly important for studying the evolution of the lunar orbit inclined to the equator and ecliptic. The best approach to this very complex problems belongs to Goldreich [19].

It should be emphasized that his method is valid only for the circular motion of satellites and cannot be extended to elliptical.

2.3. Tidal Evolution of a Circular Satellite Orbit Inclined to the Equatorial Plane and to the Ecliptic.

The method of Goldreich [19] is based on the existence of three different time scales in the dynamic evolution of the Earth-Moon system. By assumption, the Earth revolves along a circular orbit of constant radius about the Sun. All motions are considered in the geocentric system (Figure 2.9). The short time scale is determined by the periods of revolution of the Moon and Sun around the Earth, a month and

year respectively. The intermediate time scale is established by the periods of precession of the planes of the lunar orbit (X_1 X_2) and terrestrial equator with respect to the axis of the ecliptic (Y_3): at present they are 18.6 years and about 26,000 years respectively. The relative motion of these two planes is periodic (this will be shown below); this period is only slightly longer than the period of precession of the lunar orbit.

The long time scale is determined by the rate with which the tidal friction changes the state of the Earth-Moon system; at present the long scale is measured in billions of years. The rate of tidal changes is determined by the integral value of Q(t) for the Earth. The method for considering the evolution of the lunar orbit is a combination method: first an analytic averaging of the equations of motion over the short time scale is carried out, then a numerical averaging over the intermediate time scale; the averaged equations are then integrated numerically over one step back with respect to time on the long scale. The corresponding numerical parameters are found for the Earth-Moon system and substituted into the equations of motion, which are again averaged over the short and intermediate time scales /52 and again integrated numerically one step into the past on the long scale, etc. (an abbreviated derivation of the equations of Goldreich is given in Appendix 1).

Figure 2.10 presents data characteristic of the evolution of the Earth-Moon system, where the distance between the Earth and Moon r in Earth radii is taken as the independent variable. The difference in the numerical data, according to Goldreich from two different expressions for the torques from Mac Donald [5] and Kaula [18], is quite imperceptable. Not considering the solar tides could affect only the orbit of the Moon near the present. However, the differences in this region are insignificant in the two approaches.

Let us turn to the interpretation of the numerical results of Goldreich. The curves for all three angles of inclination: the terrestrial equator to the ecliptic, the lunar orbit to the ecliptic and the orbit to the equator, undergo a splitting in some region. The

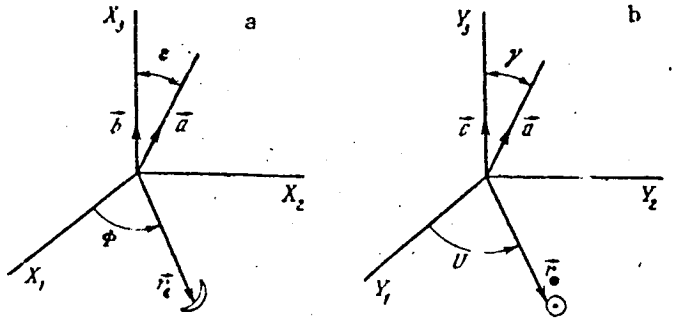

Figure 2.9. Coordinate system X_1, X_2, X_3 coupled to the lunar orbit (X_1, X_2), and the system Y_1, Y_2, Y_3 coupled to the ecliptic (Y_1, Y_2).

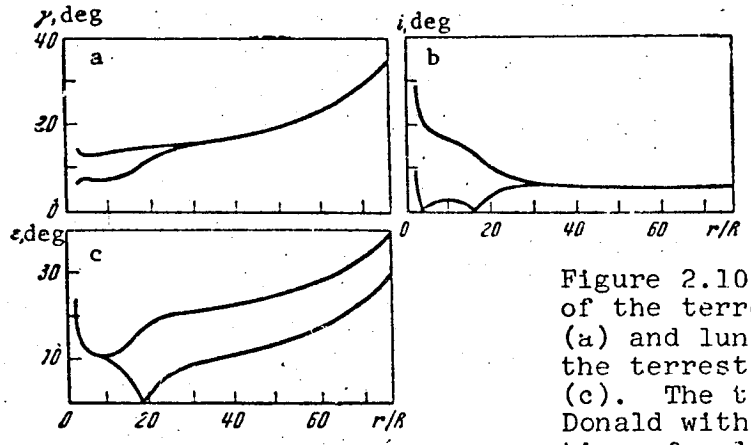

Figure 2.10. Inclination of the terrestrial equator (a) and lunar orbit (b) to the terrestrial equator (c). The torques of MacDonald without consideration of solar tides.

upper and lower branches of the curves in the region of splitting determine respectively the maximum and minimum values of the angles over the precession period, whose character changes in the course of tidal withdrawal. The theory of the precession of a satellite orbit inclined to the equator of a planet and to the ecliptic was developed by Tisserand [20]. Qualitatively, it amounts to the axis of rotation of a planet being the axis of precession of the orbit with great accuracy for near satellites, and the axis of the ecliptic being that for distant satellites. The transition zone from close to distant is rather narrow, in connection with the fact that the corresponding moments of the forces K_2 (the solar torque on the orbit of the satellite) and L (the torque from the equatorial bulge of the planet) are related as a^5. Thus, L dominates in the near zone, and K_2 in the distant. In the transition zone, the axis of precession is a straight line passing within the angle formed by the axes of the equator and ecliptic. /53

The plane perpendicular to the actual axis of precession of the orbit is called the "natural plane;" it is always located between the planes of the equator and ecliptic. The middle of the transition zone a_{crit} can be found from equating $K_2 = L$ (cf. Formula (15) in Appendix 1), which gives:

$$\left(\frac{a}{R}\right)_{crit} \cong \left[\frac{4I}{3}\frac{M}{M_\odot}\right]^{1/3}\left(\frac{a_\odot}{R}\right)^{1/3}, \qquad (2.39)$$

where M and $M_\odot$ are the masses of the Earth and Sun respectively, a and $a_\odot$ are the distances of the satellite to the planet and the Sun, R is the radius of the Earth.

With the present rotational angular momentum of the Earth $a_{crit} \approx 10$ R; with consideration of the redistribution of angular momentum in the past, with a higher rotation of the Earth and with a correspondingly greater I, $a_{crit} \approx 17$R. Of course, at the present distance of the Moon 60.3 R, the axis of precession of its orbit is the axis of the ecliptic with great accuracy, but in the past at a < 10 R this axis was the axis of rotation of the Earth. The graph for the angle of inclination of the lunar orbit to the equator indicates that nowhere in the past was it zero; it was 10° at a minimum at a distance of about 10 R. A similar conclusion was reached by us earlier [21] on the basis of the works of Mac Donald [5] and Sorokin [22], who made calculations for the two-body problem (the Earth and Moon) and thus operated with the average values of the angle. Their curves are plotted for comparison in Figure 3.5 (cf. following chapter). At closer distances the angle ε would have been significantly greater than 10° both in the cases under consideration as well as with consideration of terms of higher order in the expansion of the tidal potential [11].

This result is extremely important for understanding the past of the Moon. Firstly, it is difficult to represent the evolution of the orbit of a single whole Moon, beginning with distances closer than 10 R. The constant misalignment of the plane of the orbit of the Moon and the plane of the equator of the Earth is evidence both against its splitting off from the Earth as well as against its accumulation from a circumterrestrial swarm of small satellites in a very close orbit

around the Earth. For the part of the swarm located close to the Earth, /54 the plane of the equator of the Earth must be the plane of symmetry, just as the rings of Saturn revolve in the plane of its equator. The orbit of the Moon formed close to the Earth would have had to inherit the position of the plane of symmetry of the swarm, i.e., to revolve in the plane of the terrestrial equator, which, as we will see, never happened (cf. Figure 3.9). Secondly, the accumulation of the Moon at greater distances beginning with 30 R and farther is unlikely. The swarm revolving around the Earth in the far zone would have had the plane of symmetry coinciding with the ecliptic, and the Moon growing in this zone would also have revolved in the plane of the ecliptic.

Goldreich considers the zone between 10 and 30 R as uniquely possible for the accumulation of the Moon, since in this region the angle obtained from the calculations is approximately the same as the inclination of the "natural plane" to the equator of the Earth. Because of the inelastic collisions of the particles of the swarm, the orbits of the particles must tend to occupy positions close to the "natural plane," which in this zone is found between the equator and ecliptic. It is true that there are not the conditions here for the formation of a very thin disc (of the type of the rings of Saturn) with a sharply expressed plane of symmetry and this, in the opinion of Goldreich, is evidence not in favor of the accumulation hypothesis.
We do not consider this reason important; the order of the circumterrestrial swarm, in which accumulation of a satellite is possible, will be specially evaluated in the fourth chapter. The problems of the accumulation of the Moon and the further evolution of its orbit will be considered more broadly. We emphasize here that the important result of Goldreich is suitable for a model of the Earth-Moon system, in which the Moon is always a single body and always revolves about a circular orbit, on which act only tidal friction and the perturbing forces from the Sun and the equatorial bulge of the Earth. It is assumed that there are no cataclysms in the system, discontinuous changes in position of the terrestrial axis in space, fusion of the Moon with other satellites of the Earth or objects from interplanetary space. Meanwhile, the idea of accumulation of the planets and satellites from smaller objects certainly assumes collisions with large masses [23, 24]. Thus, the work of Goldreich can clarify the tidal

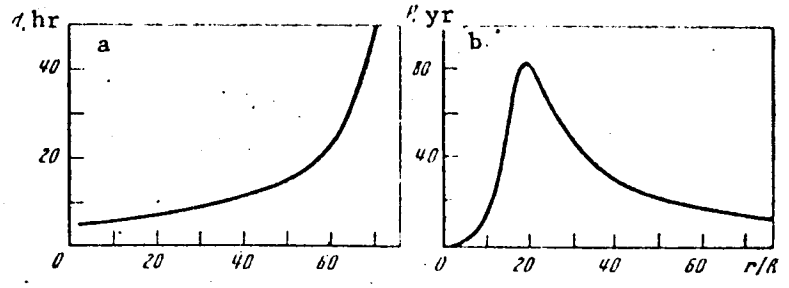

Figure 2.11. Duration of the day (a) and the period of precession of the lunar orbit (b). The torques of Darwin-Kaula with consideration of solar tides.

history of the lunar orbit only in the post-accumulation, i.e., in the geological stage of development of the Moon and Earth.

2.4. Geophysical Effects of Tidal Evolution

The position of the terrestrial axis in space and the velocity of rotation of the Earth have also changed simultaneously with the changes in the lunar orbit because of tidal friction. Figure 2.10 a shows that the axis of rotation of the Earth was previously less inclined to the axis of the ecliptic than at present, but was never collinear with it in the geological past. Tidal friction will lead /55 in the future to an increase in the inclination of the terrestrial axis. The rate of change of this angle with time is negligibly small: 1-2° per billion years. Figure 2.11 a shows the change in the duration of the terrestrial day in the course of tidal evolution. A similar picture was also obtained previously for the circular coplanar two-body problem (cf. 2.2). At the closest possible distances of the Moon from the Earth, the day equalled 5-6 hours, and the state of rotational nonequilibrium of the Earth was never obtained, for which the period of its rotation would have been 2.6 hours. This fact is also evidence against the hypothesis of the splitting off of the Moon from the Earth, as is the inclination of its orbit to the terrestrial equator. The axial rotation of the Moon at present is synchronized with the revolution, and this state is stable. It is impossible to estimate the original rotation of the Moon, since both the energy and angular momentum of this rotation is negligibly small in the total balance of the system. However, it can be stated that for any initial angular velocity of rotation less than the velocity for rotational instability,

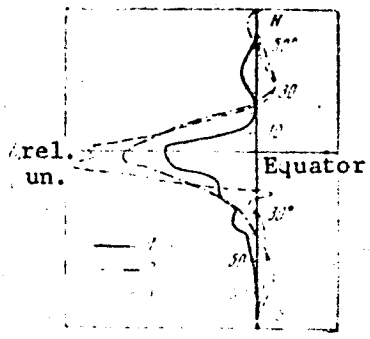

Figure 2.12. The distribution of the braking tidal torque L from waves M_2 for various latitude zones in the Pacific Ocean.

the tidal deceleration was very strong, if the Q-factor of the Moon was the same order of magnitude as for the Earth. In fact, substitution in place of the mass and radius /56 of the Earth in Formula (2.13) of the same values for the Moon and also $K_2 = 0{,}02 - 0{,}03$ gives a deceleration $d\Omega/dt$ several hundred times more effective than the deceleration of the rotation of the Earth. Thus, it can be assumed that the Moon has been in synchronized rotation during practically all geological time.

Let us turn again to the release of tidal energy in the Earth and Moon. We have already noted that the circle approximation is suitable for investigating the evolution of the lunar orbit, and to evaluate the release of tidal energy in the Earth the approximation of a circular equatorial orbit of the Moon applies, since the velocity of rotation of the Earth and, consequently, the kinetic energy of rotation have practically no difference in the case of finite inclination ε and $\varepsilon = 0$. Thus, we can assume that the integral release of tidal energy in the Earth can be determined from Formula (2.21), where the lower limit of the distance of the Moon from the Earth should be taken equal to 10 R, taking into account the conclusions of the preceding paragraph (cf. also Figure 2.6). Consequently, over the entire geological history of the Earth, the total tidal energy released in the Earth, independent of its dissipative properties, is:

$$E_{\text{tidal}} \leqslant 2.8 \cdot 10^{37} \text{ erg.} \qquad (2.40)$$

The distribution of this integral energy, firstly, between the ocean and solid Earth and secondly, between the various layers in the interior of the Earth, is of interest for geophysics.

The distribution of the tidal energy being dissipated between the ocean and solid Earth is not yet known exactly. If all the change in the velocity of rotation of the Earth, revealed from astronomical observations, are due to tidal friction, then the torque produced between the Earth and Moon is $L_2 = 3{,}9 \cdot 10^{25}$ dyne·cm, and the corresponding total release of energy in the Earth is about $2.7 \cdot 10^{19}$ erg/sec,

the parameter Q corresponding to the entire Earth is about 13. The ever greater role of the terrestrial oceans in the dissipation of tidal energy has been determined recently. In addition to the tidal friction in low-water seas and channels, oscillatory processes have been observed in the deep-water parts of the ocean, which are also "dissipators" of energy [25]. Verification of this is the direct calculation of the tidal torque from the entire water mass of the oceans, which was carried out by M. V. Kuznetsov and N. M. Pariyskiy [13, 26] from the current cotidal charts of K.T. Bogdanov and V.A. Magarik, Asad and Pekeris and Tsakhel' (curves 1 - 3, Figure 2.12). The torque is larger than the "astronomical" presented above, and is $(7-8) \cdot 10^{23}$ dyne·cm. Consequently, there must exist internal processes in the Earth accelerating its rotation (for example, the subsidence of heavier masses toward the center), in order to compensate the tidal slowing of the rotation, which for such a large torque would be greater than the "astronomical." The total magnitude of the /57
energy now being dissipated cannot be measured directly. It is derived "from the torque;" if the new data are true, then the dissipation in the oceans must exceed a value of $2.7 \cdot 10^{19}$ erg/sec. This does not imply that energy has now stopped dissipating in the solid tides. The phase lag of the solid tides in the Earth is actually significant [9]: it is the order of 1°, although it is still difficult to evaluate it precisely from the observations, which is why there is not a sufficiently certain estimate of energy being dissipated in the interior.

Analogous phenomena also exist in the other planets of the solar system: the resonance rotation of Mercury, the deceleration of the rotation of Venus, the synchronized rotation of the satellites of Mars and Jupiter, and, of course, the absence of satellites of Mercury and Venus. All these phenomena can be explained only with the presence of dissipative processes in the interiors of the solid planetary bodies without oceanic shells (cf. Chapter 6).

It is probable at present that tidal energy the order of 10^{19} erg/year is being dissipated in the solid Earth, and in those layers

or parts with lower values of Q. For the thermal properties of the entire Earth as a whole, this is only an insignificant addition to the radiogenic heat, whose release is approximately 20 times greater. However, because of the nonuniform distribution in the Earth of parts with low viscosity, in which an overwhelming part of the tidal energy is dissipated, temperature inhomogeneities can be produced in the Earth [27].

Let us consider for an example the thermal conditions of such an inhomogeneity, which is part of a plane-parallel layer in the upper mantle of the Earth, in which a greater part of the release of tidal heat is concentrated. A spherical layer of thickness ΔR has a volume of approximately $3\Delta R/R$ times less than the volume of the Earth. We assume that the effective dissipator is not the entire layer, but only approximately half of it; for example, extended centers in the low and mid-latitudes, where the tidal deformations are most significant. Then for a layer thickness of $\Delta R \tilde{\sim} 200$ km, the contribution of the tidal energy will be the same as the contribution of radioactivity, which is understood to be uniformly distributed over the entire thickness of the mantle. In the case of a layer close to the surface, one can apply the problem for the semi-infinite plane, since the solution of the problem for plane layers differs from the solution for spherical layers by no more than 10%, if the thickness of the layers does not exceed 50 km [28]. As is well known, the solution of the inhomogeneous heat conduction equation for a semi-infinite plane:

$$\frac{\partial T(x,t)}{\partial t} = k\frac{\partial^2 T}{\partial x^2} + \frac{H(x,t)}{c} \qquad (2.41)$$

with the initial and boundary conditions;

$$T(x,0) = T_1(x), \quad T(0,t) = 0, \qquad (2.42)$$

where $x = 0$ at the surface of the Earth can be represented in the form /58
of the sum of the solution for the homogeneous equation (without sources) with these initial and boundary conditions and the solution of the inhomogeneous equation [with consideration of the source function $H(x,t)$] and zero initial temperature:

$$T(x,t) = T_2(x,t) + \vartheta(x,t), \qquad (2.43)$$

where

$$\vartheta(x,0) = T_1(x), \quad \vartheta(0,t) = 0, \quad T_2(x,0) = 0, \quad T_2(0,t) = 0,$$

while:

$$T_2(x,t)=\frac{1}{2\sqrt{\pi}C_p}\int_0^t d\tau\int_0^\infty \frac{H(\xi,\tau)}{\sqrt{k(t-\tau)}}\left\{\exp\left[-\frac{(x-\xi)^2}{4k(t-\tau)}\right]-\right.$$
$$\left.-\exp\left[-\frac{(x+\xi)^2}{4k(t-\tau)}\right]\right\}d\xi. \qquad (2.44)$$

The true temperature at a given depth is the sum of the values of $T_2(x,t)$ and $\vartheta(x,t)$. However, for our purpose of determining the excess temperature related to the additional heat sources, which are "switched on" at some stage of the thermal history, it is sufficient to evaluate only the value of T_2, i.e., the temperature of the medium heated from 0° from the time of "switching on" both the principal and additional sources. The value of $\vartheta(x,t)$ reflects the thermal prehistory of the model $\vartheta(x,0)=T_1(x)$, and is also the background on which the phenomenon of interest occurs, because $\vartheta(x,t)$ denotes the temperature of the cooling medium without sources. We give the function $H(x,t)$ in the form:

$$H(x,t)=\begin{cases}H_r(t), & 0\leqslant x\leqslant h_1,\\ H_r(t)+H_e(t), & h_1\leqslant x\leqslant h_2,\\ H_r(t), & h_2<x<\infty.\end{cases} \qquad (2.45)$$

Then the integral over ξ can be expanded into three integrals with limits corresponding to (2.45). The substitution of $(x\pm\xi)/\sqrt{4k(t-\tau)}=\alpha$ gives:

$$T_2(x,t)=T_2^{(r)}+T_2^{(e)}=\frac{1}{C_p}\int_0^t H_r(\tau)\Phi\left(\frac{x}{\sqrt{4k(t-\tau)}}\right)d\tau+$$
$$+\frac{1}{2C_p}\int_0^t H_e(\tau)\left[\Phi\left(\frac{x-h_1}{\sqrt{4k(t-\tau)}}\right)-\Phi\left(\frac{x-h_2}{\sqrt{4k(t-\tau)}}\right)+\right.$$
$$\left.+\Phi\left(\frac{x+h_1}{\sqrt{4k(t-\tau)}}\right)-\Phi\left(\frac{x+h_2}{\sqrt{4k(t-\tau)}}\right)\right]d\tau, \qquad (2.46)$$

where Φ is the error integral.

This formula is convenient in that the first term on the right side ($T_2^{(r)}$) describes the heating from 0° of a homogeneous model with uniformly distributed radioactive sources $H_r(t)$, and the second term ($T_2^{(e)}$) determines the temperature correction caused by the tidal heat evolution $H_e(t)$ in the layer between h_1 and h_2. The value of this correction remains unchanged and above the total temperature in which $\vartheta(x,t)$ appears. If one now sets $H_r(t)$ = const and $H_e(t)$ = const, which

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is practically true for the past billion years, then one can integrate (2.46) and obtain the formula for calculation in the form:

$$T_2(x,t) = \frac{H_r}{C\rho}\left[\left(t+\frac{x^2}{2k}\right)\Phi\left(\frac{x}{\sqrt{4kt}}\right)+\frac{x\sqrt{t}}{\sqrt{\pi k}}e^{-x^2/4kt}-\frac{x^2}{2k}\right]+$$
$$+\frac{H_r}{2C\rho}\sum_i(-1)^n\left[\left(t+\frac{x_i^2}{2k}\right)\Phi\left(\frac{x_i}{\sqrt{4kt}}\right)+\right.$$
$$\left.+\frac{x_i\sqrt{t}}{\sqrt{\pi k}}e^{-x_i^2/4kt}-\frac{x_i^2}{2k}\right], \qquad (2.47)$$

where $x_1=x+h_1$, $x_2=x+h_2$, $x_3=|x-h_1|$, $x_4=|x-h_2|$ and n takes even or odd values in the cases

	$i=1$	$i=2$	$i=3$	$i=4$
$x \leqslant h_1 \leqslant h_2$	even	odd	odd	even
$h_1 \leqslant x \leqslant h_2$	even	odd	even	even
$h_2 \leqslant x$	even	odd	even	odd

Figure 2.13 presents the solution for $T_2(x,t)$ with $H_e/C\rho = H_r/C\rho = 10^{-14}$ deg/sec; for C = 0.2 cal/gm·deg and ρ = 3.5 gm/cm^3 this corresponds to $H_e = H_r = 0.7 \cdot 10^{-14}$ cal/cm^3-sec, i.e., a release of heat typical for chondrites. The value of H_r in this case produces a homogeneous background, and its numerical value does not effect the value of the temperature excess. A lower value of H_r is usually taken for the mantle in calculations of the thermal state of the Earth (taking into account the depletion of radioactive elements in the mantle). The heat conduction coefficient k was taken as 0.01 cm^2/sec. In the numerical example under consideration, the "dissipation layer" is taken close to the surface (h_1 = 100 km, h_2 = 300 km). Thus, the entire zone under consideration lies in a region of intense heat outflow, which decreases all excess thermal effects. In spite of this, the excess temperature in the layer (h_1, h_2) and its vicinity has reached 30° over 100 million years and 100° over one billion years, becuase of the distribution of the excess heat in the adjacent layers. This estimate is not a maximum; the excess temperature could have reached even higher values, if the "dissipation layer" had been deeper. It is only important here that the mechanism for additional heating act over a sufficiently long time. The viscosity coefficient and the relaxation time depend very strongly on temperature [29]: at a temperature of the material of 1600 - 1800°K typical for depths of 200-300 km, its change by 100° leads to a change in the viscosity coefficient of 10 /60

times. The viscosity coefficient also depends very strongly on the properties of the crystalline structure of the material of the mantle, which can differ from place to place at the same depth. These circumstances must evidently cause a variability of the viscosity coefficient within the layer where it reaches minimum values. This can also lead to the creation of "centers" of tidal energy dissipation in the Earth and to the enhancement of initially small temperature inhomogeneities due to the excess heating in the "centers." Tidal heating of the interior of the Moon, as was already noted above, is insignificant (cf. 2.3 of this chapter).

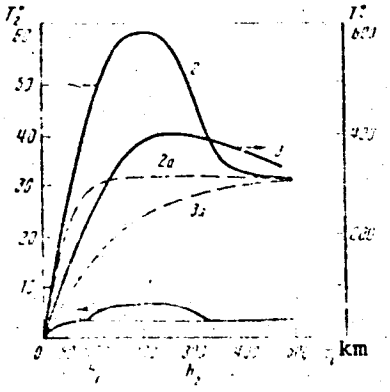

Figure 2.13. Heating of the upper layers of the Earth with a homogeneous distribution of radioactive elements (dashed lines) and additional (tidal) heat sources in the layer (h_1, h_2) (solid line). Curves 1-3 are the temperatures of the upper layer of the Earth 10^7, 10^8 and 10^9 years after the beginning of the heating.

The consideration of the tidal evolution of the lunar orbit shows that the Moon was closer to the Earth in the past and was a more normal satellite. The eccentricity of the orbit and its inclination to the equator of the Earth were less than the present, although the equator and orbit were never coplanar. This imposes a limitation on the value of the initial distance of the Moon from the Earth. The Moon could not have split off the Earth, and also could not have formed as a single body at distances less than 10R and greater than 30 R.

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CHAPTER 3

VARIOUS HYPOTHESES ON THE ORIGIN OF THE MOON

The age of the Earth from geochemical estimates coincides with /62
the age of the oldest stony meteorites, which equals 4.6 billion years [1]. On the basis of measurements of the ages of lunar rocks, the conclusion has been reached that the age of the Moon is 4.5 - 4.6 billion years [2]. The coincidence of the epochs of formation of all bodies, which have been investigated up the present by chemical methods, verifies the generality of the process leading to the generation of bodies of the planetary system. Until the direct determinations of the ages of celestial objects, this general character of the process was postulated from the dynamic peculiarities of the solar system.

In spite of such good agreement between mechanics and space chemistry in the approach to planetary cosmogony in general, the specific method of formation of the Earth and Moon has not been interpreted directly from the observational data extracted at the surfaces of these objects. The expectation that the investigations of the surface of the Moon could unlock the secret of the origin of the terrestrial planets was not justified. The surface of the Moon appeared after investigations in the form in which it was formed during the period from 3.9 to 3.1 billion years ago, since later processes did not change it very much. But the early stages of the history of the Moon were erased. In order to "complete" the past of the Moon and to describe the first 0.1 - 0.2 billion years of its history, it is necessary to "filter out" the many factors superimposed in the initial period of its existence. The work of many scientists will be devoted in the next years to this very complex problem of solving the evolution of the interior and surface of the Moon.

Consideration of the tidal evolution of the lunar orbit brings additional information for explaining the history of the Moon, by limiting the region of possible "starts" for the motion of the Moon

around the Earth. The development of the model of the formation of the Moon can be compared with the cutting of an underground tunnel toward the tunnel which is being "dug" by analyzing present observational data. Both tunnels will join in the favorable case of constructing the correct model for the formation of the Moon. At present there exist three hypotheses for the formation of the Moon: 1) In orbit around the Earth; 2) By splitting off from the Earth; 3) By capture by the Earth from a circumsolar orbit. We will consider them in turn. /63

3.1. Formation of the Moon in a Circumterrestrial Orbit.

Model of the Institute of Earth Physics. The formation of the Moon in the form of a separate body in circumterrestrial orbit is a component part of the process, which, according to the idea of O.Yu. Shmidt [3,4], converted the circumsolar gas-dust cloud into the system of planets. The scheme of Shmidt for the formation of the planets was developed on the basis of physical and mechanical considerations for the purpose of explaining the basic dynamic characteristics of the solar system and also the evolution of the Earth as a planet. The authors of works [5,6] were then led to similar ideas on the basis of physical-chemical and nuclear-physical data. The priority in the development of the theory of O.Yu. Shmidt belongs to the Institute of Earth Physics (IFZ) AN SSSR. Co-workers of the Division of Earth Evolution, created by Shmidt in 1946, continue to work on this problem at present in the Institute. The model of the formation of the Earth from the preplanetary cloud was developed in quantitative form by V.S. Safronov [7]. The first attempts were recently made at a direct computer simulation of several cosmogonic processes, previously studied by the methods of gas dynamics or statistical physics. The evolutionary scheme of the "machine" preplanetary cloud into the system of planets [8] actually repeats the scheme of V.S. Safronov.

The theory of O.Yu. Shmidt in its present quantitative form is evaluated as one of the most promising. In 1972 the French Academy of Sciences conducted a special international symposium in Nice, devoted to a critical comparison of the observed data with the existing models of the origin of the solar system. The model of Safronov was

one of the six selected for consideration (the other five were the models of Alfven, Hoyle, Schatzman, Cameron, ter Haar). It encountered essentially no criticism, whereas the other models were seriously criticized [9]. Kaula evaluates the model of Safronov as dynamically the most valid picture of the evolution of the preplanetary cloud into the system of planets with the minimum activity of the Sun in the past and with the minimum mass of the cloud [10].

We will briefly discuss the sequence of events in the preplanetary cloud in accordance with the model developed at IFZ AN SSSR. Whatever the origin of the preplanetary cloud: its capture by the Sun or separation from the protosun in the process of compression, it is natural to assign to the cloud a "cosmic" or "solar" composition, i.e., 99% hydrogen and helium and 1% of the other elements in the same proportion as in the Sun. This assumption agrees with the fact that the two /64
largest planets, Jupiter and Saturn, which now have 95% of the mass of all the planets, consist mainly of hydrogen and helium. The further development of the preplanetary cloud must have gone according to its internal law, conforming to the attraction of the Sun and its radiation. Other stars and galaxies could not have significantly affected the solar system, since its dimension (within the orbit of Pluto) is ten thousand times smaller than the distances to the nearest stars. The temperature determined by the radiation of the Sun must have been established in the preplanetary cloud, since there were no natural sources of energy in the cloud. The mass of all the planets is now 0.13% of the mass of the Sun. The mass of the cloud must have exceeded the mass of the planets, since it was composed of volatile gases, which do not appear in the planets. But if it is assumed, as was done, for example, by Cameron [9, page 56], that the mass of the cloud was at first much greater than 0.05 or 0.1 solar mass, say, equal to 2 or 3 of its masses, then very great difficulties arise in explaining the ejection of the excess from the solar system.

Moreover, a massive cloud would evolve most rapidly not into a system of planets, but into a star-like satellite of the Sun [8]. For a small (as compared to the Sun) mass of the cloud, the average spatial density of matter in the cloud was very small, comparable to the density of laboratory vacuums. Under these conditions no substance

could be in the liquid state: the existence of only two phases - solid and gaseous — were possible. After the luminosity of the Sun decreased to its present value, a temperature of about 0°C must have been established at the distance of the Earth from the Sun after a comparatively short time (the order of 10^6 years). Thus, all metals and water in particular must have been in the solid state. Only the strongly attenuated radiation of the Sun reached to the more remote region of Jupiter and Saturn, and the temperature was probably below -200°C. All the low-melting and volatile compounds - water, methane, ammonia, carbon dioxide, etc. — must have been in the frozen state here. But nowhere within the preplanetary cloud (as in the interstellar clouds) did the temperature fall so low that hydrogen and helium could be frozen into solid particles. Thus, the preplanetary cloud consisted of the gases of hydrogen and helijm and of dust particles, whose composition was increasingly enriched with volatile substances with increasing distance from the Sun. The role of gas pressure in the cloud was small. The basic force controlling its motion was the gravitation of the Sun, and the cloud revolved about it with the Keplerian angular velocity practically throughout.

The further course of events depends on whether there were large-scale vortex (turbulent) motions in the cloud or its revolution around the Sun was laminar. In the case of turbulent motion, the gas and /65
dust are mixed, the cloud has the form of a thick torus without a significant separation of these two components. The growth of solid objects is possible only by the combination of individual particles with their collisions. The formation of massive gaseous planets is excluded here. However, large-scale vortex motions could not be maintained in the cloud longer than 10 million years [11]; they must have been attenuated. The dust settled to the central plane, and the cloud took the form of a gaseous torus with a thin equatorial layer of small solid bodies and particles. There could not have yet been any sort of large bodies among them. This stage was not necessarily traversed "simultaneously" in the entire preplanetary cloud, but the tendency toward the collection of the dust component into a thin disc was everywhere.

During this "flattening," the relative velocities of dust particles were small, probably no more than several meters per second. Under

these conditions, adhesion of silicate particles with each other is possible, whereas metallic particles, as a rule, do not adhere [12]. Besides the direct growth of bodies by adhesion, the formation of dust clumps of mixed composition is possible in the thin dust layer because of its gravitational instability. The collision of the clumps leads rapidly to their combination with each other, consolidation and conversion into continuous solid bodies with cross sections of tens or hundreds of kilometers. The mutual attraction of large bodies distorted their ordered motion along circular orbits about the Sun. The velocities of objects with collision increased to several hundred meters or even several kilometers per second. This aided the "welding" of metallic (mainly iron) particles to each other and, conversely, led to the fracturing of small silicate bodies. Such a difference in the behavior of iron and silicates is observed over a wide range of "cosmic" velocities - from 1.5 to 10 km/sec [12, 13]. The oxides of metals, iron in particular, are also broken up with collisions with "cosmic" velocities, but the indications are that the fracture fragments are somewhat larger than, for example, for the oxide of silicon - the principal component of stony bodies (cf. Chapter 5.3). The fact that the collisions of particles are always inelastic, with a loss of part of their kinetic energy to heat, fracturing, deformation and ejection of fragments, is very important for planetary cosmogony.

The study of the distribution function of many cosmic bodies subjected to collisions with very different velocities is an extrordinarily difficult problem. The result of the collision (adhesion or fracturing, the dimensions of the final products) depends on many factors: the velocity at impact, the composition of the bodies, the shape of the bodies, the monolithicity. There are laboratory data in the required range of velocities only for metallic and stony bodies. There are no laboratory data at all on collisions for bodies containing volatile components. Thus, only some average approach to the problem is possible. It is somewhat simplified by the fact that the principal factor controlling the process of coagulation in the preplanetary cloud is the mass of the bodies. /66

Large bodies, independent of composition, have the tendency for growth, and the intermediate and small are broken up and gradually scooped up by the large. If follows from the theory that with collisions of solid bodies a mass distribution is established in their ensemble, which is well represented by an inverse power law $N(m)dm \sim m^{-q}dm$ where $q \approx 1{,}8$ [14, 15]. A significant regularity is observed for this value of q: a larger part of the material (mass) is included in the large bodies, while the larger fraction of the total surface belongs to the fine component of the ensemble. The power law is violated only at the ends of the distribution: for the smallest and largest bodies. Particles with dimensions less than 10^{-5} - 10^{-6} cm are subjected to additional effects: light repulsion, evaporation because of the large curvature of the surface, and their role is insignificant in the general ensemble. On the contrary, the largest body in the ensemble plays an extremely great role. The tendencies for growth are such that this body "separates" from the competitors by several orders of magnitude in mass and becomes the single nucleus of a planet. In the zone of the Earth for example, its nucleus, "body No. 1", was during the active stage of growth approximately 10^3 times more massive than "body No. 2," which in the final analysis was absorbed by it. The separation of the preplanetary cloud into autonomous "supply zones" of individual large nuclei resulted from the distribution of mass and velocities of the preplanetary bodies. The larger the "surface" density of the material (i.e., the mass of a column with unit cross section perpendicular to the central plane of the cloud) at a given distance, the larger the preplanetary bodies here, the more massive the nuclei and the wider its supply zone. In the limit when the nucleus is large (say, several times more massive than the present Jupiter), the entire preplanetary cloud can become a single supply zone for a star-like satellite of the Sun. The transition from planet to stellar object, whose radiation is maintained by nuclear synthesis reactions, is only quantitative. Dwarf stars are known with masses of at most 0.03 $M_\odot$, i.e., 30 times larger than the mass of Jupiter.

The "surface" density σ in the preplanetary cloud is usually defined as the addition of material to the mass of the planets required to coincide with the solar composition. Thus, in the region of the

terrestrial planets σ could have reached $5 \cdot 10^3 - 10^4$ gm/cm^2, but only 10 - 20 gm/cm^2 occurs in the fraction of condensed material. Hydrogen and helium must have been dissipated under the effect of the solar corpuscular flux and did not appear in the composition of the preplanetary bodies. The hydrodynamic effect of gas on the preplanetary bodies could not have been significant, since its density did not exceed 10^{-9} gm/cm^3. In the region of Jupiter and Saturn, σ would have been about 100 gm/cm^2, if the present composition of these planets is exactly the "solar" composition and nothing was lost from their zone. However, it is more likely that σ was several times greater than the present, since part of the material in the form of preplanetary /67
bodies was ejected from the zones of Jupiter and Saturn by perturbations from the massive nuclei of these planets [16]. The growth process of these nuclei was aided by the great abundance of ice and other frozen material in the preplanetary bodies. When the nuclei of Jupiter and Saturn reached approximately the mass of the Earth (this was, in a way, a "critical mass"), they must have been actively enveloped by an atmosphere of hydrogen and helium. Consequently, in the zone of the Earth, whose mass is close to "critical," the gas must have already been dissipated before the beginning of active accretion - the combination of gas to the nucleus.

Accretion is an accelerating process, for which the increment of mass of a planet is proportional to the square of the mass; thus, once started, it goes to the complete scooping up of gas in the zone, overtaking the ejection of gas by solar particles. Only the accretion of gas can explain the present composition of Jupiter and Saturn, which, judging from their low density, must be very close to the composition of the Sun [17]. Bodies of significantlylower mass than critical in their zone have quite another composition: the satellites of Jupiter and Saturn, the largest of which are comparable in mass to the Moon, have average densities of about 2.5 - 3.0 gm/cm^3, which indicates their stony-ice composition.

The average density of Uranus and Neptune indicates that their composition cannot be "solar." Hydrogen and helium appear only in the composition of the atmospheres of these planets. The most suitable substance for them both in cosmic abundance as well as in density

(for the corresponding pressures in the interiors of Uranus and Neptune) is water with various additions [18]. This fact has very great significance for understanding the processes at the far reaches of the preplanetary cloud.

Uranus and Neptune were formed much more gradually than the terrestrial planets and also Jupiter and Saturn. This is explained by two causes: the slow scooping up of material because of the slower revolution of Uranus and Neptune around the Sun and the lower density of the material. Over the time of growth of Uranus and Neptune up to critical mass, the hydrogen and helium had essentially left the solar system.* Moreover, many solid preplanetary bodies must have left the region of Uranus and Neptune; such is the result of the gravitational perturbations from the larger nuclei of the planets at the periphery of the planetary system, where the gravitation of the Sun is weak. If there had not been at first in the zone of Uranus and Neptune in addition 10 times more condensed material, then these planets simply could not have grown to the present dimensions over the whole time of existence of the solar system (more than 10^{10} years would have been necessary [20]). Calculations indicate that the ejected preplanetary /68 bodies from the region of the Jovian planets must have frequently reached the Oort cloud of comets, which is at the very boundaries of the sphere of influence of the Sun [16]. It was discovered only comparatively recently that the nuclei of comet consist mainly of H_2O ice with admixtures [21]. The "aqueous" composition of Uranus and Neptune, confidently predicted theoretically, agrees well with the hypothesis about the origin of comets as preplanetary bodies ejected by perturbations - frozen samples of the material of Uranus and Neptune.

The concept of O.Yu. Shmidt, developed at IFZ AN SSSR, explains from a single point of view the origin of the basic physical and dynamic peculiarities of the planetary system. In this concept the formation of the satellites is thought of as a process accompanying the formation of the planets. O.Yu. Shmidt did not successfully investigate this

* The problem of the dissipation of gases from the solar system has not been studied quantitatively to a sufficient degree, since the intensity of corpuscular radiation of the early Sun is not known. The dissipation mechanism is developed in [19].

process in quantitative form himself. However, the general idea was rather clearly expressed in his book "Four Lectures on the Theory of the Origin of the Earth" (1957).[4]: "During the formation of the planets, in the process of convergence of particles with the large nuclei of the planets, some of the particles lost so much velocity in colliding, that they dropped out of the general swarm and began to revolve about the planet. Thus, a clump is formed about the planetary nucleus - a swarm of particles revolving about it in elliptical orbits. These particles also collide and change their orbits. The same processes, as for the formation of the planets, will occur on a reduced scale in these swarms. A majority of the particles falls to the planet (combine with it), part of them will form a circumplanetary swarm and combine into independent nuclei - the future satellites of the planet ... With the averaging of the orbits of the particles forming the satellites, the latter acquires a symmetric, i.e., close to circular, orbit lying in the equatorial plane of the planet."

A theoretical model of the formation of the Moon, which will be discussed in detail in Chapters 4 and 5 and its application to the formation of the satellites of the planets in Chapter 6, was developed on the basis of this idea by us at IFZ AN SSSR, beginning in 1960.

We want to note in this chapter the fundamental importance of the prerequisites of the growth of the Earth for the various models of the formation of the Moon. Calculation of the growth of the Moon according to the model developed at the IFZ AN SSSR leads to a duration for the growth of the order of 10^8 years (in more detail, cf. Chapter 4). As will be noted later, the authors of other models of the formation of the Earth arbitrarily assign a shorter duration, 10^5 [22] or 10^6 years [23]. Such a short scale for the formation of the Earth was recently used in [24]. In some schemes of Cameron, the time of growth of the Earth is postulated even shorter - 10^4 - 10^5 years [25]. It should be emphasized that the time of growth of the Earth cannot be a free parameter; it results from the requirement of the whole concept.

The Öpik model. In the literature of recent years, one frequently /69
encounters the expression "Öpik model" as applied to the origin of the Moon. It amounts to the formation of a swarm of satellites around the Earth in the form of a ring of the type of the rings of Saturn. The process of accumulation of the Moon from such rings occurs rather rapidly - over 10^2 - 10^5 years. Such an assertion was made in qualitative form in 1961 [26]. In 1971 Öpik [27] acknowledged the identity of his concepts with the previously proposed concept of O.Yu. Shmidt, which we had developed further. Öpik noted that the formation of the Moon from a satellite swarm is the only acceptable way: "It should be emphasized that the direct condensation of the Moon from the gaseous state is a rather unlikely assumption. Even if the required conditions of extremely low temperature and high gas density could have been accomplished, the Earth would have used them in the first place and then turned into a Jovian planet. The accumulation of solid particles is the only way by which the Moon could have been formed. The velocity of fall onto it must not have exceeded 11 km/sec, otherwise mass loss would have been obtained instead of accretion, even for the present lunar mass; for the growth of a smaller mass a lower velocity limit should have been applied to 2 km/sec and lower. Hence, the inevitable requirement that the accumulation took place from some ring of solid particles, in which the relative velocities would have been small" [27, page 136].

The logic of the arguments which led Öpik to this conclusion was independent from that which is the basis of the works of O.Yu. Shmidt and his coworkers. Öpik thoroughly analyzed the present observational data for the small bodies of the solar system and from them reconstructed the past of the Moon. In his many and very detailed works devoted to the Moon, he considered the structure and evolution of its surface; he contributed many new ideas to the theory of impact crater formation, significantly strengthening its position. The extraordinary breadth of investigations of the bodies of the solar system - from meteors to the Jovian planets - made possible his complex approach to the study of the Moon on the basis of all the available observational data. Some of his predictions have been brilliantly confirmed. For example, Öpik estimated from meteoritic investigations the addition of meteoritic material into the lunar

regolith as 2 - 3% [26], while chemical analysis of the samples brought back to Earth have given for this addition 1.5 - 2% [24]. With the present level of knowledge about the size distribution function for small bodies of the solar system, this coincidence should be considered very good.

Öpik took his own original path to his concepts about the origin of the Moon in a near orbit about the Earth. He formulated the problem in 1961: to collect from the statistics of the ellipticity of the oldest craters on the visible side of the Moon the traces of larger tidal deformation than that which corresponds to the present distance /70 of the Moon from the Earth. On photographs taken from the Earth, he measured the ratio $(a- b)/a$ for 53 highland craters within an angular distance of 12° from the center of the visible disc of the Moon and for 125 craters in the region between 50 and 70° from the center. He eliminated the ellipticity related to the projection of the lunar sphere onto flat film. The remaining ellipticity must consist of two components: a random component caused by the inclined fall of a crater-forming body flying with limited velocity and a systematic component caused by the tidal deformation of the Moon (because of terrestrial tides) and the rotation of the Moon about its axis. Öpik obtained for the random component an average ellipticity of about 0.070 for the central region and 0.096 for the region close to the limb, from which he included with consideration of the low profiles of the craters that the craters were formed by impacts with low velocities the order of 3 km/sec into material with a shear modulus 5 - 10 times lower than for basalts. One can conclude from the value of the velocity that the falling bodies were in circumstantial orbits.

Öpik found for the systematic component of ellipticity in the central region a certain predominance of the maridional distribution, which could correspond to tidal deformation of the Moon at a distance of (4.5 ± 0.4) R. For the region close to the limb the distribution of ellipticity best corresponded to a distance of (8.1 ± 1.1) R. Hence, the conclusion follows that the formation of the Moon was completed when the Moon was in an orbit with a radius of (5-8) R and later bombardment could no longer erase the old relief from the surface.

It should be noted that the accuracy of the observational data, which led Öpik to such an important conclusion, was insufficient nevertheless. Kaula [28] noted the desirability of repeating the investigation of the ellipticity of craters with more current material, that is, with the help of photographs of the Moon from spacecraft oriented from the stars.

The difference of the Öpik model from ours is mainly that he does not relate the formation of the prelunar ring of satellites necessarily with inelastic collisions of particles in the vicinity of the growing Earth. Öpik also considers such possibilities as the escape of material from a rapidly rotating Earth and the formation of a satellite ring first within the Roche limit, and then tidal withdrawal of the individual satellites (there must have been no less than 1000) beyond this limit and the gradual accumulation of the Moon.

However, in this case, even if the duration of the tidal withdrawal of small satellites (the time of withdrawal is inversely proportional to the mass of the satellites) could be increased to the necessary degree, the Moon would have had an equatorial orbit, which does not agree with the facts.

Another possibility, according to Öpik, is related to a close /71
passage of a large body (the protomoon) into the immediate vicinity about the Earth and the disruption of this body by the tidal forces of the Earth. (Mitler [29] recently expressed a similar hypothesis.) In this case, solid fragments of the body with radius up to 200 km could have withstood the disrupting effect of the tidal forces and, having first been within the Roche limit, then could have left it and begun to collect into the Moon. In both cases, the formation of a circumterrestrial swarm due to collisions is not required. However, both possibilities considered by Öpik illustrate the necessity of the formation of the Moon from a circumterrestrial swarm of particles, rather than confirming the hypotheses of splitting off or capture.

Another difference of the Öpik model from ours consists of the assumption of a shorter scale for the growth of the Earth, $4 \cdot 10^5$ years, which is less valid (cf. above).

The Ringwood model. In 1960 Ringwood expressed a hypothesis about the formation of the Earth with the simultaneous precipitation of the iron core, calling it the "single-stage" hypothesis [30]. He assumed the initial components to be solid particles and bodies with dimensions in the interval from one to 10^6 cm and with the composition of the carbonaceous chondrites. The duration of the accumulation was assumed short (about 10^6 years). In 1966 Ringwood proposed that a by-product of such an accumulation could be the formation of the Moon from the primary massive and hot atmosphere of the Earth, from which a disc of solid particles would be separated with cooling, in the form of precipitates, containing less iron and more volatile components than the material of the Earth: the so-called "precipitation hypotheses" [31]. When it was first revealed in 1969-1970 that the Moon is depleted in volatile components [24], Ringwood introduced a correction to this hypothesis and cited the elimination of the most volatile substances from the primary atmosphere of the Earth with the help of an intense corpuscular radiation of the Sun, which, according to the assumption, was then a T Tauri star.* In 1970 [23] his model took the following form.

Without justifying the brevity of the scale of accumulation of the Earth, Ringwood assumed that this process occurred at a temperature of from 1000 to 2000°C. The sequence of physical-chemical conversions, which occurred according to Ringwood, is easily seen in Figure 3.1.

The pattern qualitatively amounts to the development in the process of the growth of the Earth of a thick primary atmosphere of CO, H_2 and evaporated silicates, which prevented the escape of heat with accumulation. The Earth is formed as if "from the inside out:" inside; /72
chondrite material, from the outside - reduced metals. Such a configuration is unstable: the metal (iron) collapses into the core, because of which the temperature of 2000°C is reached with the release of additional energy. In the atmosphere of the Earth is from 20-50% of its mass. With expansion and cooling, the atmosphere condenses into

* T Tauri Stars, similar in mass to the Sun, are usually related to interstellar dust clouds. A significant discharge of material takes place from these stars in the form of charged particles fluxes - "stellar wind," which exceeds by 6 ot 7 orders of magnitude the present "solar wind."

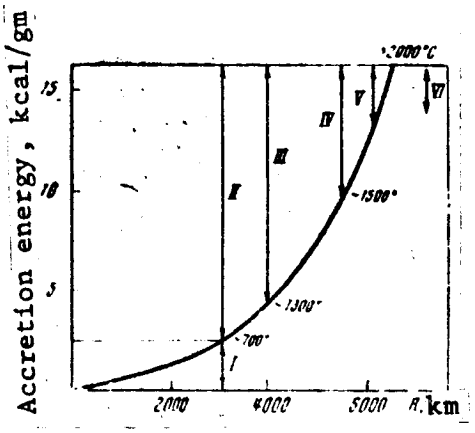

Figure 3.1. Sequence of the physical-chemical conversions of the Earth with rapid accumulation.

I - Temperature limit of existence of a cold oxidized core enriched with volatile elements;
II - Region of formation of the primary atmosphere (planestesimals are degassed with impacts) and the reduction of oxidized iron into metallic;
III - Region of selective evaporation of Na, K, Rb, Cs, Pb, Bi, Tl, In, etc. into the primary atmosphere;
IV - Region of selective evaporation of SiO, Mg;
V - Region of complete evaporation of accumulated planetesimals;
VI - Region of melting of iron and silicates forming the core.

a ring of planetesimals around the Earth, from which the Moon later forms.

With the expansion of the atmosphere, precipitation occurs in an order opposite to that shown in Figure 3.1. The first condensates, precipitating at a higher temperature, form large bodies - up to 10^7 cm. The volatile components condensing later most likely form fine particles, which are carried away by the gas of the primary terrestrial atmosphere being blown away with flares of the Sun in the T Tauri stage. The nonvolatile calcium and aluminum silicates, iron, $MgSiO_3$ appear predominantly in the composition of the Moon. The sorting of the volatile compounds occurs mainly because of the earlier precipitation of the nonvolatile compounds, which formed the larger particles (the planetesimals). The volatile elements Na, K, Rb, Cs, Zn, Hg, Cd, Cu, Ge, Pb, Tl, Bi, In, Ga, Sb, As, Br, J, Cl, Se and Te are extremely depleted in the Moon. Ringwood assumes that the oxidation of iron was not complete; the partial melting of the mixture FeO plus silicates permitted precipitation of the metal, causing a deficiency of the siderophylic elements and a low ratio of Fe^{3+}/Fe^{2+} with the presence of a certain amount of metallic iron. Ringwood assumes a time for the accumulation of the Moon from the circumterrestrial ring as 80 years at most, referring to Öpik [26]. Of course, with such a rapid accumulation of the Moon from small particles, all their gravitational energy /73 could not be radiated into space, and the upper layers of the Moon are heated almost to melting by the end of its growth.

The hypothesis of Ringwood has a qualitative nature; only the general chemical regularities are considered in it. Its deficiency is the neglect of the mechanics of the process. The time for the accumulation of the Earth is assumed without any dynamic justifications whatsoever. Its value of 10^6 years is selected by Ringwood as some average between the scale of Hanks and Anderson (10^5 years) and the scale of Safronov (10^8 years). The least justified, from the dynamic point of view, is the formation of the circumterrestrial ring of satellites from the extended atmosphere. By atmosphere is usually meant the gaseous shell of a planet, which is held by its gravity and which rotates together with the planet. To precipitate from the atmosphere a ring of satellites formed with Keplerian angular velocity requires a great reserve of angular momentum for the whole atmosphere. Since up to 50% of the mass of the entire Earth was transported to the atmosphere, it is natural to conclude that the entire Earth formed according to Ringwood must have been rotating with the Keplerian angular velocity at the equator, i.e., at the limit of rotational stability. One can evaluate at once the angular momentum of the protoearth (H_{pe}): with its dimensions greater than the present, this momentum would have been no less than that which would correspond to the rotation of the present Earth with the Keplerian angular velocity at the equator, i.e., with a period of 1.5 hours.

Thus, for the protoearth separated from the primary atmosphere of the Moon:

$$H_{pe} \geq H_{crit} = I_{crit}\omega_{crit}, \qquad (3.1)$$

where I_{crit} is the moment of inertia of the Earth, corresponding to a rotation with a velocity ω_{crit}. Substituting $I_{pres} = 8.09 \cdot 10^{44}$ CGS $< I_{crit}$ and $\omega_{crit} = 12.6 \cdot 10^{-4}$, we obtain:

$$H_{pe} > 10 \cdot 10^{41} \ CGS. \qquad (3.2)$$

It is well known that the total angular momentum of the present Earth-Moon system is $3.45 \cdot 10^{41}$ CGS. Consequently, the Earth-Moon protosystem must have discarded into space an excess momentum, twice as great as all this total momentum. The ejection can be accomplished only with matter, if there are sufficient forces. However, in the scheme of Ringwood only the volatile components are lost from the Earth-Moon protosystem from the primary composition of the type of carbonaceous

chondrites. The content of volatile components could be somewhat greater than in the Earth as a whole, but in all they are only several percent by mass, and not 2/3 by mass, as would be required for the loss of angular momentum.

There is another difficulty of the hypothesis of Ringwood, which is general for all hypotheses for the splitting off of the Moon from the Earth. This is the position of the lunar orbit in the equatorial /74 plane of the Earth, which contradicts the calculations of the tidal evolution (cf. preceding chapter). Moreover, not one of the planets of the solar system, including the Jovian, rotates at the limit of rotational instability: rotation is always several times less, and simulation of the rotation of the growing planets leads to approximately the present periods, and not to the much shorter periods corresponding to rotation at the limit of rotational instability [32 - 35].

The model of Anders. After observing a deficit of volatile and low-melting and an excess of high-melting elements in lunar basalts as compared to terrestrial basalts, Anders with co-workers [24] were led to the conclusion that a model of the formation of the Moon in a circumterrestrial orbit should be considered best from the point of view of the chemical composition. He evaluated the so-called "average temperature of the accumulated material" for the Moon. It was equal to 650°K, while the analogous "temperature" for the Earth was 90-100°K lower, i.e. 550°K. The concept of this "average temperature" rests on cosmogonic ideas, which are discussed in detail in the works of Larimer and Anders [36, 37] and also in article [38].

According to the assumption, all the basic chemical compounds in the terrestrial planets and in meteorites arose at one epoch, when the hot gaseous preplanetary cloud cooled from an initial temperature of about 2000°K, not having natural sources of heat. Even a very massive cloud with a mass the order of the mass of the Sun spends no more than 10^6 years in this cooling. The composition of the cloud is assumed "cosmic;" the gas pressure (mainly determined by the abundance of atomic hydrogen) in the region of the asteroids, from which meteorites arrive at the Earth, is assumed between 10^{-3} and 10^{-6} atm. This interval of pressures encompasses both the model of the massive and the non-

massive preplanetary cloud. Then one can calculate the temperature for each specific substance. This was done in the works of Larimer and Anders.

If one now takes some complex conglomerate of substances, then one can calculate for it some "average temperature" of accumulation, which is defined as the average condensation temperature for the most abundant components, but certainly with consideration of the content of the volatile components. According to the concept of Anders, they are the basic "cosmic temperatures" of the mixture. In this sense, the "average temperatures of accumulation" were evaluated for the Earth and Moon as 550 and 650°K.

Later, Anders [24] reached the conclusion that such a difference in the temperatures speaks in favor of the formation of the Moon in a near orbit about the Earth. The logic of the arguments is thus: the Earth is accumulated rapidly on a cosmogonic scale, i.e., at a stage in cooling of the preplanetary cloud, when its temperature had not yet fallen to the values determined by the radiation of the Sun. At the /75
last stage of growth, already having a satellite - the Moon — the Earth absorbed the lower-melting substances than the Moon, because the Moon is moving in a near (5-10)R orbit around the Earth with a geocentric velocity of about 3 km/sec and thus has a smaller "capture cross section" for particles from the zone of growth of the Earth containing the volatile components. Being in a further orbit (60-100)R, the Moon would have had a smaller geocentric velocity (<1 cm/sec) and would have captured more volatile elements than necessary.

This argument in favor of the formation of the Moon in a near orbit does not seem convincing. The difference in the chemical composition of the Earth and Moon, being formed in one zone, can be explained differently (cf. Chapter 5). The mechanics of the growth of the Earth and Moon according to Anders requires an artificially short time scale ($\sim 10^6$ years), where this short time interval must certainly coincide with the other, too short interval for the cooling of the cloud.

3.2. Formation of the Moon by Splitting off from the Earth*

The hypothesis about the origin of the Moon by splitting off from the Earth arose at the end of the 19th century.

The calculations of G. Darwin of the tidal evolution of the lunar orbit showed that the Moon must have been significantly closer to the Earth in the past. As was noted in the preceding chapter, the initial orbit cannot be determined from such calculations, one can only eliminate the unlikely configurations. The accuracy of the calculations of Darwin was not great, and even the "distance of closest approach," which exists in the two-body problem, was not found. Nonetheless, the hypothesis of the splitting off of the Moon from the Earth was expressed by him. Darwin [42] noted that if both bodies were merged, preserving the momentum of the Earth-Moon system, then this mass would rotate with a period of about 4 hours. The period of the semidiurnal wave of the solar tides would then have equalled 2 hours. By assuming that the fundamental period of the natural oscillations of the Earth is also about 2 hours, Darwin concluded that resonance must have arisen and the height of the tidal bulge must have increased continuously until a piece of material (the Moon) split off from the Earth. However, Jeffreys [43] showed that the internal friction must have prevented the tidal bulge from developing to the required dimensions, so that the rupture because of resonance is unlikely. Moreover, the fundamental period of the natural oscillations of the Earth is less than 1 hour (54 minutes), and not 2 hours, as assumed by Darwin.

Attempts have been made to justify the hypothesis of the splitting /76
off of the Moon from the Earth with the help of the mechanics of rotating liquid masses. The fact is rather well known that there are errors in the works of Darwin and Jeans devoted to the gradual splitting of a rotating liquid mass into two components because of rotational instability, which were later discovered by A.M. Lyapunov and Cartan [44]. These works referrd mainly to homogeneous rotating bodies. The idea about rotational instability was developed in the works of James [45]

*Many critical investigations describe the different versions of this hypothesis. In this paragraph we will use the conclusions from works [24, 28, 39 - 41], etc.

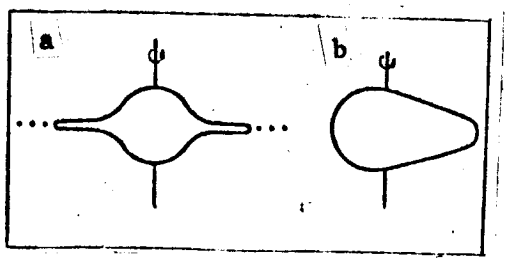

Figure 3.2. Rotational instability for: (a) $I \leqslant 1/4MR^2$ — escape from the equator, (b) $I \geqslant 1/4\ MR^2$ — pear-shaped figure.

as applied to polytropic structures, i.e., gaseous or liquid bodies with a spherically symmetric distribution of density ρ related to the pressure p by the formula:

$$p = k\rho^{1+1/n},$$

where n is the polytropic index; the larger n, the greater the density concentration toward the center of the body. James found that the rotational instability appears differently depending on n. For $n > 0.8$, when the dimensionless moment of inertia $I/MR^2 < 0.28$, escape of material from the equator in a thin disc must occur with the achievement of critical angular velocity of rotation, as shown in Figure 3.2 a. For $n < 0.8$, and accordingly $I/MR^2 > 0.28$, the instability develops through a series of triaxial ellipsoids, then being converted into a pear-shaped figure, from which the massive bulge splits off (Figure 3.2b).

The first form of instability does not apply to the Earth in its present form, since $I_\oplus/MR_2 \approx 0.33 > 0.28$ at present. By assuming the unlkely event that with its formation the Earth was blanketed by a thick primary atmosphere and that its angular momentum exceeded the present momentum of the Earth-Moon system by 3 or more times, the value of the dimensionless moment of inertia could have been decreased to the required limit. But in this case, the escape of material from the equator would have led to the separation of at most about 1/1000 of the mass of the central body, which is clearly insufficient for the formation of such a large satellite as the Moon.

The second type of instability, from the condition for the moment of inertia, could have applied to the Earth with its present density distribution, if it had been liquid and rapidly rotating. However, /77
the bulge splitting off is very massive: 10 - 20% of the mass of the central body. It is usually represented as a consequence of a wave disturbance in the form of a third order harmonic superimposed on the triaxial Jacobi ellipsoid, which rotates about its shortest axis. The "neck" develops along the line of the angles of the spherical harmonic.

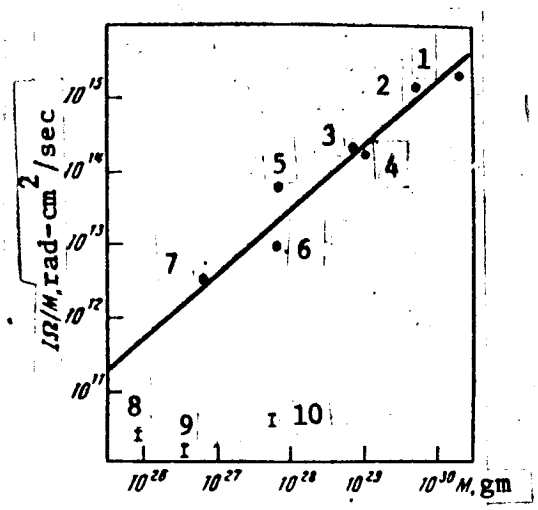

Figure 3.3. Dependence of the specific angular momentum of the planets $I\Omega/M$ on their mass.
KEY: 1- Jupiter; 2- Saturn; 3- Uranus; 4- Neptune; 5- Earth-Moon; 6- Earth; 7- Mars; 8- Moon; 9- Mercury; 10- Venus.

In order to split off, the massive bulge must extend to a distance exceeding the Roche limit. The tidal stability of the newly formed "drop" is unclear. Along with the problem of the ejection of excess mass (about 10 lunar masses) there remains the problem of the release of the excess angular momentum, which exceeds the momentum of the Earth-Moon system by more than 3 times.

Actually, for a total momentum of the Earth-Moon system exceeding the present value by 3 or 4 times, the specific angular momentum of the protoearth (i.e., the value of $H_\oplus/M_\oplus$) would have been no less than $2 \cdot 10^{14}$ rad·cm^2/sec, which would have strongly segregated the Earth from all the planets, for which is observed a certain empirical dependence between the mass and specific momentum [39] (Figure 3.3). According to this dependence, the specific momentum of the protoearth could not have exceeded $2 \cdot 10^{13}$ rad·cm^2/sec.

Moreover, the formation of a disc or the splitting off of a pear-shaped bulge in the plane of the equator of the central body results naturally from rotational instability. However, calculations of the tidal evolution show that at no time in the past was the lunar orbit coplanar with the terrestrial equator.

The hypothesis of the splitting off of the Moon from the Earth, based on similar mechanics, was considered in works [30, 31, 46, 47].

Attempts have also been made to find a source of energy for the separation of the Moon from the Earth. Bullen [48] relates the splitting off of the Moon from the mantle of the Earth with an assumed instantaneous phase transition of the silicate material of the terrestrial core into the metallic state (this hypothesis has not yet been verified experimentally). Under certain assumptions about the

physics of the transition, it must be accompanied by the instantaneous release of tremendous energy - $3 \cdot 10^{38}$ ergs. Such energy would have been overly sufficient for the splitting off and conveyance of the Moon from the surface of the Earth to its present distance. However, Bullen generally ignores the mechanics of rotation. Moreover, this energy could not have communicated the required angular momentum to the /78 ejected mass and placed it into a circular orbit. We have already mentioned in the second chapter that the lunar orbit has been close to circular both at present and in the past.

In spite of the many difficulties with the hypothesis of the splitting off of the Moon from the Earth, attempts have continued until recently to revive it in varied form. The strangest application of the mechanics of rotating masses is the hypothesis of Lyttleton about the origin of the Moon, which was expressed by him in the article [49]. Lyttleton had previously given in his book [44] a valid critical analysis of the mathematical errors and ambiguities, which were present in the hypotheses about the formation of double stars by the division of rotating liquid masses. Now Lyttleton, in turning to the Earth-Moon system, indicates the impossibility of the separation of a single rotating mass into two bodies with a relative mass of the components of 81:1, as for the Earth and Moon. He proposes a "corrected" version of the splitting off hypothesis. In this version, the original liquid mass divides with the rotational instability into the planets Earth and Mars with a ratio of the components 10:1. From the "neck" between these masses is collected a spherical "drop" - the future Moon, which falls in some obscure manner into a circumterrestrial orbit, while Mars and the Earth occupy their proper place in the solar system. The hypothesis of Lyttleton is completely unconvincing from the point of view of celestial mechanics: the orbits of the Earth and Mars are almost circular, they intersect nowhere, nothing indicates their separation from a single mass.

The idea that the Earth and Moon were completely molten at some time is inconsistent with the geophysics and physics of the Moon. The Earth would then have stratified and tectonic motions would have ceased, it would have become "tectonically dead" [50], which is not the actual case. The same could be said about the planet Mars, where there

is undoubtedly tectonic activity [51]. The Moon, although close to a state of "tectonic rest" at present, also could not have been completely molten at some time, because its basalts neither in age nor in chemical composition could be products of a single magmatic source (cf. Chapter 1).

The close similarity of the average density of the Moon (3.34 gm/cm^3) and the density of the rocks of the upper mantle of the Earth (3.3 - 3.5 gm/cm^3) is usually the cause for the revival of the hypothesis of the splitting off of the Moon from the Earth. This coincidence is not random. Enormous areas on the surface of the Moon are filled with basalts similar in type to the terrestrial, and at depths of tens of kilometers there are probably rocks similar to the ultrabasic rocks of the mantle of the Earth; the velocity of seismic waves indicates this (cf. Chapter 1). However, the differences in chemical composition of the lunar and terrestrial basalts are such that they could not have been the products of the same thermal and chemical evolution, as would have been if the Moon were a piece of the mantle, having split off after the formation of the terrestrial core. Firstly, these differences are /79 revealed in the content of low-melting (volatile) substances (cf. Figure 1.9), in which the Moon is greatly depleted as compared to terrestrial rocks. The depletion evidently occurred even before the accumulation of the Moon. It is difficult to imagine the loss of such heavy elements as Pb, Bi, Tl (atomic mass of $\sim$ 200) from the surface of an already formed moon. Even if under forces of increased volatility, the atoms of these elements were evaporated from molten parts of the lunar surface, then because of the large mass they would have to have settled rapidly again somewhere on the surface of the Moon, and not escaped into space. And of course, it is not possible to explain by any selective dissipation from the surface of the Moon the enrichment of its rocks with high-melting elements Ti, Hf, Y, Sc, Zr, etc.

Secondly, the differences of the lunar rocks from the terrestrial appear in the ratios of the siderophylic elements, which in geochemical processes are like satellites of iron (Co, Ni, Pt, Ir, Os, Re, Pd, Rh, Ru, Au). Gold in lunar basalts, for example, is 10^2 times less than in terrestrial, and the other siderophylic elements compose lunar rocks in "nonterrestrial" proportions (cf. Figure 1.8). If the composition of the Moon were already formed in the interior of the mantle of the

Earth, when the iron accumulated in the terrestrial core, then the content of siderophylic elements in the Moon and the terrestrial mantle would have been the same. The great differences in the relative compositions of these elements indicates the dissimilarity of the chemical evolution of the Moon and the mantle of the Earth.

We see that neither the mechanics of rotating masses, nor study of the tidal evolution of the Earth-Moon system, nor data about the chemical composition of the Moon verify the hypothesis of the splitting off of the Moon from the Earth. It represents only a certain historical interest.

3.3. Capture of the Already Existing Moon by the Earth

The hypothesis of Urey and its logic. Proposals about the capture of the Moon by the Earth [52] are constructed on purely chemical arguments. They arose in connection with the difficulty in explaining the formation of these bodies from a single supply zone, since in this case, according to Urey, the Earth and Moon would have had identical chemical composition.

Ideas about the chemical composition of the Moon were constructed in the 40's and 50's only from data about its average density. The observed average density of the Moon equals 3.34 gm/cm^3; referring to standard temperature and pressure it would be 3.38 - 3.41 gm/cm^3. These values of density are still significantly lower than the values of 3.5 - 3.7 gm/cm^3, typical for chondrites, which at the beginning of the 50's were considered the prototype of the nonvolatile fraction of the preplanetary material. The carbonaceous chondrites are now considered the more likely prototype.

The cause of the differences in density of the Moon and chondrites, /80 according to Urey, must be the relative depletion of iron in the Moon: 11 - 14% by weight as compared to 22 - 29% by weight in chondrites. Another possibility would be the enrichment of volatile components in the Moon for an equal iron content. If one attributes to the Moon the average content of the most iron-depleted chondrites (the so-called LL-group), one would have to add about 10% by weight graphite or 2-3%

by weight water. But Urey already considered it more correct to start from the differences in the iron content. The Moon was much more deficient in iron than the chondrites, Mars (28% iron, according to Urey), and the Earth with an iron core (28% iron, according to Urey, and up to 35%, according to modern estimates). The hypothesis of Ramsey about the silicate composition of the terrestrial core would have only brought the composition of the Moon closer to the composition of the Earth, but all the same the terrestrial planets would have differed in composition, if one takes into account that Mercury is the richest in iron, while Venus contains no less than the Earth with an iron core.

Thus, the Moon appeared to be a "strange" object both with respect to the terrestrial planets and with respect to the asteroids, which are the precursors of meteorites. At that time (the beginning of the 1950's) the amount of iron in the nonvolatile fraction of the material of the Sun was evaluated as 6% at most with a probable error of approximately 2 times, i.e., it was significantly less than in meteorites. Urey reached the conclusion that the Moon is closest of all to the Sun according to the amount of iron. Hence, the conclusion followed that the Moon was first formed as an object of solar composition, i.e., it included more than 99% of H, He, C, N, O and only less than 1% of all the other elements. Such an object must have had by necessity a significant mass: $2.2 \cdot 10^{28}$ gm, i.e., 3 or 4 times more massive than the Earth.

In the same year, Urey proposed a cosmogonic hypothesis, according to which there were the conditions for a gravitational instability in the preplanetary cloud with a mass greater than 1/3 of the mass of the Sun, and this led to the disintegration of the cloud onto individual objects with mass a hundred times greater than the Moon. The typical mass of the primary objects in the zone between the ring of the asteroids and Jupiter was just $2.2 \cdot 10^{28}$ gm. Urey called them "moon-like" objects. According to his assumption, these bodies collided with each other, lost the volatile components and were converted into objects of stony composition. At this stage of collisions, one of the "moon-like" bodies was captured into a circumterrestrial orbit in some undefined manner.

The difficulties of this concept are obvious. It is very difficult to imagine (even with consideration of mutual collisions) that an object with a mass intermediate between the mass of Earth and Uranus, i.e., a rather massive planet, could lose the volatile components and be converted into a Moon-like object (in the direct sense, and not in the sense of Urey). The sorting of gases from a self-gravitational body is a process, extremely slow and ineffective on cosmogonic time scales, as was shown by I.S. Shklovskiy [53] and Opik [54]. One can find a /81 statement in a later work of Urey, which indicates that he understands the difficulties with the dissipation of the volatile elements from the primary objects: "It could be that I am lazy, but Mother Nature is not lazy; but I prefer the loss of gases up to the formation of strong gravitational fields" [55]. The subsequent fate of the great number of remaining primary objects is also unclear in the model of Urey.

There remains to be added that present re-evaluations of the iron content in the Sun toward an increase by several times (up to 7 times) eliminated one of the important arguments from the hypothesis of Urey: the assumed peculiar closeness of the composition of the Moon to the composition of the Sun.

A second, and no lesser, difficulty is related to the mechanics of capture of a Moon-like body into a circumterrestrial orbit. Let us consider this problem in more detail, following [56]. Urey himself does not indicate a specific scheme for capture; however, one may think that it is in the form of gravitational capture. The formulation of the problem about the capture of a "moon-like object" by the Earth in the original sense of Urey is unacceptable. In having a mass of 3 or 4 times greater than the mass of the Earth, such an object with capture would have become the central body itself, or merged with the Earth into a single mass.

Let us consider the probability of capture by the Earth of the Moon in its present form. Such a capture can be accomplished in principle with the approach of three bodies: the growing Earth and two bodies similar in mass to the Moon. The approach of the two "moons" must have taken place within the gravitational sphere of Hill, surrounding

the Earth*, and the relative distance of the "moons" for the transition of one of them into circumterrestrial orbit must not have exceeded their "gravitational" radius. The latter is defined as the "target" distance, at which the velocity vector of one of the bodies turns by an angle the order of $\pi/2$. The dispersion of the velocities of the preplanetary bodies within the zone of the Earth was determined by the perturbations of the largest body - the growing Earth - and was the order of $v=\sqrt{Gm/\theta r}$, where m and r are the mass and radius of the Earth and $\theta \tilde{\sim} 3$ is a numerical factor [7]. Even for $M_{pres}/2$ the dispersion of velocities of bodies within the zone is 3 - 4 km/sec and thusly exceeds the parabolic velocity at the surface of the moon-like body (for the Moon this velocity is 2.4 km/sec). If it is also taken into account that, according to the assumption of Urey, the Moon came from somewhere in the belt of the asteroids, then it must have had a geocentric velocity "at infinity" the order of 10 km/sec; i.e., the encounter of the two "Moons" must have occurred with a relative velocity necessarily greater than the parabolic velocity at their surface. Under these conditions, the gravitational radius of the bodies is less than their geometric radius. Gravitational capture into an almost circular orbit is then accomplished only under very special conditions of approach. Hence, it follows that the probability of capture of a finished Moon was less than the probability of its collision with a body similar to it. /82

We will show that the probability of one such collision over the entire time of growth of the Earth was less than 10^{-2} even if all the preplanetary bodies in the zone of the Earth were of lunar dimensions and each of them could have become the present Moon. The probability of capture of a body coming in from outside the zone of the Earth was even less.

Actually, the number of collisions of bodies of dimensions a_1 per unit volume per second is $2n^2\pi a_1^2\bar{v}$, where $n = \rho/m_1$ and all a_1 and m_1 are the same. The spatial density p is determined by the amount of material not yet scooped up by the Earth in its supply zone, i.e., M_{pres} - m

* The gravitational sphere of Hill is defined as the maximum closed surface, within which stable (according to Hill) motion of the satellites of a planet is possible. Its radius is approximately $\mathfrak{R}\sqrt[3]{m/3M_\odot}$. Here $\mathfrak{R}$ and $M_\odot$ are the distance from the Sun and its mass (cf. Chapter 4).

and the volume of this zone. We take the ordinary boundaries of the zone from 0.8 to 1.3 AU; the uniform thickness of the zone $H = \pi\Re\bar{v}/2v_c$, and v_c is the circular heliocentric velocity at this distance: $v_c = \sqrt{GM_\odot/\Re}$. Hence:

$$\rho \cong \frac{2(M_{pres} - m)v_c}{3\pi\Re^2\bar{v}}. \tag{3.3}$$

The number of collisions per second in the Hill sphere equals:

$$\frac{dN}{dt} = 2\frac{\rho^2}{m_i^2}\pi a_i^2 v \Re^3 \frac{m}{3M_\odot}. \tag{3.4}$$

By dividing dN/dt by the rate of increase in mass of the Earth (according to [7], $dm/dt = 2\sqrt{GM_\odot/\Re^3}r^2(1 + 20)\rho H$, cf. Chapter 4), we obtain the total number of collisions of bodies of dimension a_i over the time of growth of the mass of the Earth from m_0 to M_{pres}:

$$N = \int_{m_0}^{M_{pres}} \frac{(M_{pres} - m)\sqrt{6\delta_\oplus}}{2\pi^2\delta_i^2(1 + 20)\sqrt{3\pi}\sqrt{G_\odot\Re^3}} dm, \tag{3.5}$$

where δ_i is the average density of the bodies, $\delta_\oplus$ is the average density of the Earth. For a_i = 1000 km (Moon-like size) and mass $M_{pres} = 6 \cdot 10^{27}$ gm, the number of collisions, independent of the initial mass of the Earth m_0, is less than 10^{-3}. Thus:

$$N < 10^{-3}, \tag{3.6}$$

i.e., over the entire time of growth of the Earth, one single collision in its sphere of influence of Moon-like bodies, under the maximizing conditions that all the preplanetary bodies were identical and comparable in mass to the Moon, could have been accomplished with a probability less than 0.1%. The probability of a purely gravitational capture into circular orbit of the Moon coming from the belt of the asteroids is several orders of magnitude less. /83

If one keeps the simplifying assumption that the dimension of all the particles in the supply zone of the Earth are the same, i.e., a_i = const, then Formula (3.5) is a clear illustration of the fact that the frequency of collisions of particles near the Earth increases with their decreasing dimension a_i:

$$N \sim 1/a_i^4.$$

This simple relation indicates how much simpler it is to capture the Moon "by parts" than as a whole. Exactly the same is also valid for various dimensions of the particles. The model of the formation of the

Moon from a circumterrestrial swarm is based on just this principle. A detailed discussion of this model is given in Chapters 4 and 5.

Hypotheses about the capture of the Moon by the Earth on the basis of calculations of the tidal evolution of the Earth-Moon system. The hypothesis about the capture of the Moon by the Earth, supposedly based on calculations of the tidal evolution of the lunar orbit, has become widely known thanks to Alfven [57], who stated it in qualitative form.

The idea of this hypothesis consists of the Moon moving independently of the Earth around the Sun and being captured by the Earth into an extremely elongated orbit greatly inclined to the equator, which then because of tidal friction became a close circular orbit with a radius equaling the Roche limit of 2.89 $R_{\oplus}$. The Moon then experienced partial disruption because of the tidal forces of the Earth. One part of the fragments later fell again to the Moon forming the maria and craters, and the other fell onto the Earth forming the terrestrial crust. Further tidal evolution of the lunar orbit permitted the Moon to withdraw to the present distance. The hypothesis that the terrestrial crust was "dumped from the Moon," and not separated from the interior of the Earth by differentiation, contradicts the data of geophysics and geochemistry, and it is impossible to take it seriously. However, the surprising evolution of the orbit of the Moon was actually borrowed by Alfven from the calculations of the German astronomer Gerstenkorn [58], although the capture of the Moon itself did not follow from this work, and the partial disruption at the Roche limit was not specified at all, since the calculation was performed for a constant mass of the Moon.

The work of Gerstenkorn of 1955 [58] was the first attempt to investigate the tidal history of the Earth-Moon system after the classical works of Darwin at the end of the 19th century. It attracted attention thanks to the hypothesis of Alfven and became very popular. Both the numerical conclusions from this work as well as the concept of capture were then disputed in print [39, 59 - 63], but Gerstenkorn insisted on its correctness [64], and only in 1968 did he acknowledge errors in several of his estimates [65]. A work of Gerstenkorn [66] was published in 1969, in which the formulation of the problem was

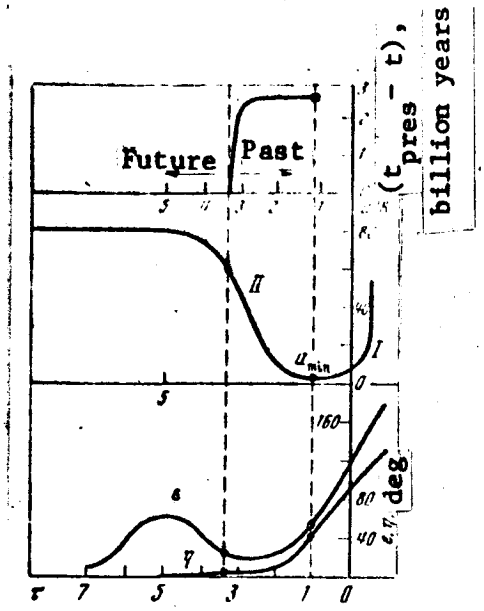

Figure 3.4. Dependence of the time t and the characteristics of the lunar orbit a, ε and η on the parameter τ.

changed (complicated). Its numerical results significantly differ from the results of work [58], making the capture of the Moon even less likely. /84

Discussion in print of the hypothesis of the capture of the Moon with references to Gerstenkorn was so wide that it would be difficult to give a complete bibliography of all these publications. The capture of the Moon was even sometimes called the "Gerstenkorn event" [67] and all possible geological and even paleontological consequences were considered. The work of Mac Donald [39] contained a criticism of the capture according to Gerstenkorn and introduced his own concept about the origin of the Moon from several protomoons at a distance of (30-40) $R_\oplus$ from the Earth. Nonetheless, evidence in favor of the results of Gerstenkorn [58]* were found even in this work of Mac Donald. These calculations are now of purely historical interest. They played a role of no small importance in stimulating a whole series of new investigations.

The results of the calculations of Gerstenkorn [58] are presented in Figure 3.4, where the parameter τ plays the role of time. The curve for a(τ) consists of two branches: the branch of approach (I) to a_{min} and the branch of withdrawal (II). It is noteworthy that a_{min} coincided here with the Roche limit equalling 2.89 R for a liquid satellite with the density of the Moon. The orbit of the Moon on the branch of approach is greatly inclined to the equator (ε > 45°); at first this is a parabolic orbit (e≅1) with retrograde motion and with a perigee of 26.1 R. Extension of the calculations into the past for τ < 1.06 was evidently considered at the time by Gerstenkorn as pure formalism, confirming the fact that the orbit of the Moon only touched the dangerous distance, but did not come inside it. In the work, he called the

* Work [58] is discussed in detail in Russian in [68, 69].

interval from the present time to the time of closest approach, the "past right up to the initial distance."

However, real significance was later ascribed to the portion of the curve formally preceding a_{min}, and it became considered as the branch of the tidal evolution following after the time of capture [57]. In recent years, such opinions can be encountered in the works of Singer [70].

The concept of capture on the basis of similar calculations is vulnerable for the following reasons.

1. The separation of the evolution curve a(t) into two branches and its singularity at the point a_{min} are a typical feature of the evolution of a circular or nearly circular orbit of a point satellite, which has a finite inclination ε to the equatorial plane of the planet, in the two-body problem. We have already noted in Chapter 2 that for an inclined orbit a_{min} plays the role of a limit of the tidal evolution, but, in contrast to a_{min} for a coplanar satellite for which the rotation of the satellite is synchronous and stable, revolution at a_{min} is unstable for $\varepsilon \neq 0$. The disappearance of the difference $(\Omega \cos \varepsilon - \omega)$, where $\Omega \cos \varepsilon$ is the component of the angular velocity of the planet parallel to the axis of the satellite orbit (the axis $\chi_3 \| \bar{h}$), denotes the cessation of the transfer of momentum in the plane of the satellite orbit, and hence the disappearance of da/dt, but with the maintenance of the transfer of momentumin the perpendicular direction (the c component of the torque related to $\Omega \sin \varepsilon$), i.e., for $d\eta/dt \neq 0$ and $d\varepsilon/dt \neq 0$. The angle η between the plane of the satellite orbit and the plane perpendicular to the invariant momentum vector of the system then changes according to the formula of Mac Donald [43]: /85

$$\frac{d\eta}{da} = -[\sin \varepsilon B(q)]/[2a(\cos \varepsilon - \alpha) F(q)], \qquad (3.7)$$

where:

$$\alpha = \omega/\Omega, \quad q = [\sin^2 \varepsilon/(1 + \alpha^2 - 2\alpha \cos \varepsilon)]^{1/2},$$
$$B(q) = \frac{1}{q^2}[F(q) - E(q)] + F(q),$$

F(q) is the complete elliptic integral of first kind, E(q) is that of the second kind. At the point $\cos \varepsilon - \alpha = 0$, $d\eta/d\alpha = \infty$, the orientation

of the satellite orbit experiences an abrupt change and the value of da/dt changes sign: withdrawal changes into approach. (Such behavior could be attributed in principle to any satellite-planet system, and not only the Earth-Moon system, if the effect of solar tides and solar precession can be neglected.)

The accuracy of the numerical calculations is not great here. The different accounting for the finite eccentricity of the lunar orbit, the properties of the Moon as an object and not as a material point, or the differences in the values of a_{min} and ε, or even a difference in the technique for calculation can give significant differences in the behavior of the "incoming" curve. This is illustrated by comparing the results of Gerstenkorn [58] and Mac Donald [39], who calculated the "incoming branch" of the tidal evolution under similar physical assumptions (although he then stated reasons against its reality). The numerical values for ε of Mac Donald differed from that of Gerstenkorn by 10 degrees. If one speaks about the capture of the Moon, then the "initial" orbit of Mac Donald is obtained not with retrograde motion, but only with a large inclination to the equator (about 33°) and a perigee distance of not 26.1 R, but 2.78 R.

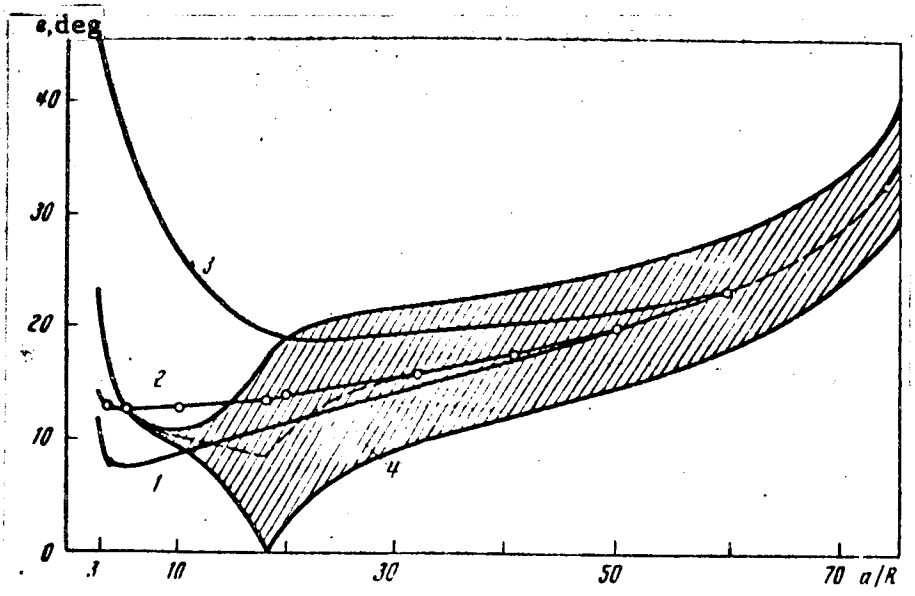

Figure 3.5. Dependence of the inclination angle of the lunar orbit to the terrestrial equator on the Earth-Moon distance from the data of various authors: 1— Mac Donald [39]; 2— Sorokin [59]; 3 — Gerstenkorn [58]; 4— Goldreich [41].

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By using very small perigee distances and large (up to 0.5 - 0.6) values of e, Singer [70] obtained an "incoming" branch with $\varepsilon < 45°$, i.e. an orbit with direct motion. All this indicates how unreliable the conclusions about the "incoming" branch of the tidal evolution of the Earth-Moon system are.

2. To justify a more confident evaluation of the "incoming" branch, one should direct particular attention to the accuracy of the calculation of the "outgoing" branch, i.e. the present branch, on which there is only a single observed point - the present state of the Earth-Moon system. All the investigators of the past states of the system have used this point as an initial condition. However, the authors of [59 - 63], while using the same physical scheme, have not confirmed the results of Gerstenkorn with respect to $\varepsilon(a)$ for past epochs. The data for $\bar{\varepsilon}(a)$ from all the mentioned sources are plotted in Figure 3.5 on the background of the Goldreich band for $\varepsilon(a)$. The oldest data (according to Darwin) also fall in the Goldreich band. All the results, except for the data of Gerstenkorn, satisfactorily agree with the results of Goldreich, whose formulation of the problem was more complete. The discrepancy in $\bar{\varepsilon}(a)$ leads to a difference in the estimate of the distance a_{min}, whose role is particularly great in the capture concept. The relation of a_{min} and ε is seen from simple relations [61]. Let us write them out.

The condition of conservation of momentum of a two-body system in a scalar form can be written in the form: /87

$$[(Mm/(M+m))a^2\omega + C\Omega\cos\varepsilon]^2 + [C\Omega\sin\varepsilon]^2 = K^2. \quad (3.8)$$

From Kepler's third law:

$$\omega = [G(M+m)/a^3]^{1/2}. \quad (3.9)$$

Setting $\omega = \Omega\cos\varepsilon$ and substituting (3.9) into (3.8), we obtain an algebraic equation with respect to a:

$$[MmG^{1/2}/(M+m)^{1/2}]a^{1/2} + C[G(M+m)]^{1/2}a^{-1/2} = [K^2 - (C\Omega\sin\varepsilon)^2]^{1/2}. \quad (3.10)$$

Its roots are a_{min} and a_{max}. Only the first is of interest. Substituting into (3.10) the value Ω at the point a_{min}, which is $3.5 \cdot 10^{-4}$ CGS according to the data of [39, 58], 59], and considering the increase of C by 5-8% corresponding to faster rotation, we obtain for $\varepsilon = 45°$, 7

(according to Gerstenkorn) [58] a_{min} = 2.89 R, and for ε = 10 - 12° (according to the data of [39, 59, 71]) a_{min} = 2.42 - 2.44 R. Consideration of the solar tides changes the value of a_{min} only slightly: according to Gerstenkorn, it would have been 2.86 R, according to Sorokin — =2.40R. Mac Donald estimated a_{min} = (2.78 - 2.72) R in work [39] and 2.50 R in a refined calculation [71], where the term P_e of the tidal potential was also taken into account along with the term P_2. These values are all somewhat greater than those found from formula (3.10), which can be considered as the standard. In 1968 Gerstenkorn [65] acknowledged that "inaccuracy slipped into" his previous calculation [58] and that after correction one should assume $\varepsilon \approx 10°$, a_{min} - 2.50 R.

Thus, after the refinements it was found that the distance of closest approach for the circular two-body problem (and probably also with consideration of the effect of a third body - the Sun) lies inside the Roche limit (2.89 R) for a liquid satellite with the density of the Moon. Consideration of cohesive forces in the solid Moon must somewhat decrease this limit.

By making use of the equations of Jeffreys [72] for the tidal stability of a small satellite and taking his critical value for the difference of the principal stresses $\Delta p_0 = 10^9$ dyne/cm^2, one can evaluate the Roche limit for a completely solid Moon as 2.80-2.81 R [61]. For a_{min} = (2.40 - 2.50) R the tidal stresses in the Moon will be approximately 2 times greater than the critical. We emphasize that a_{min} should be considered as the minimum distance not for a single approach, but over the tidal time scale (cf. Chapter 2.3). Hence, the conclusion follows that the capture concept with a parabolic orbit requires the revolution of the Moon over the course of many (>10^3) revolutions closer than the limit, where it must be disrupted by the tidal forces of the Earth. And if the violation of the integrity of the Moon is unavoidable, then the junction of the "incoming" branch with the "outgoing" branch at the point a_{min} loses all meaning, since this /88 is possible only for a single point mass.

3. However, the greatest difficulty of the capture concept is not only in the imprecision of the "branches" of the curves or even in the value of a_{min}. In the final analysis, if there had existed an

effective capture mechanism with a single passage about the Earth, then a large body with the mass of the Moon could have been captured, drawn into the Roche limit, broken up into fragments (for example, into a thousand pieces, as was assumed by Öpik [27]), and then collected again into the Moon, since the fragments, in withdrawing independently from the Earth, could move beyond the Roche limit and again combine. But it is the gravitational capture of the Moon as a whole, which represents the greatest difficulty. We have shown above the inefficiency of such capture because of collisions at the stage of the existence of preplanetary bodies. Let us now discuss the role of tidal friction.

If one turns to the first work of Gerstenkorn [58], then one can calculate the tidal change in the orbital parameters a and e with one passage of the Moon from "infinity" to a given perigee so that its orbit be converted into an ellipse in the geocentric system, by making use of equations of the type (2.32). It is found that for a perigee distance of 26.1 R for $e_0 = 1$, the decrease in e due to tidal friction in the Earth is 10 orders of magnitude less than that required for capture, and at 2.8 R is 5 orders of magnitude smaller [68]. Thus, tidal friction in the Earth only is extremely ineffective.

The work of Gerstenkorn of 1969 [66] is devoted to finding conditions for which the capture of the Moon is possible nonetheless. The basic change in the formulation of the problem, as compared to the first work, is consideration of tidal friction in the Moon itself, which, as we have seen before (cf. Chapter 2.3), can have a significant effect on the eccentricity of its orbit, if the dissipation of energy in the Moon is large. A numerical approach to the problem is also applied here. The initial conditions were not taken at present, but at an earlier epoch, when the Moon was at a distance of 10 R. This was done in order that the precession of the lunar orbit under the effect of the forces of the Sun could be neglected and to consider the two-body problem.

The new results show that the capture of the Moon could have taken place, if the dissipation of tidal energy in the Moon would have been 6 times greater than in the Earth $Q_{\mathbb{C}} = Q_{\oplus}/6$, and if the Moon had approached

the Earth several times so that the distance between the centers of both bodies at the perigee of the orbit was (1.4 - 1.6) R. These conditions imply that, firstly, the Moon must have been all in a semimolten state, otherwise such low Q_{α} are impossible. Secondly, it must have been deformed into an extremely extended body (as also the Earth), so that at perigee they both must have been touching surfaces. Gerstenkorn points out that the passage of the Moon inside the Roche limit lasts at most about one hour each time, and thus the disruption cannot /89 occur because of insufficient time. However, one can object that the fundamental period of the natural spheroidal oscillations for the Moon is at most 15 minutes [73], and thus the time for the disruption of the Moon is quite sufficient. Thus, capture of the Moon as a whole is unlikely. Kaula [28] also refers to the capture hypothesis as improbable by its very essence("inherently improbable").

The "new wave" of hypotheses on the capture of a finished Moon. Recently, in connection with the observed relative enrichment of the Moon with high-melting aluminum and calcium silicates and also in connection with its depletion in iron and volatile elements, a new wave of hypotheses about the capture of the Moon has arisen [70, 74 - 77]. From the assumption about the difference in the p, T-conditions for condensation, and then the accumulation of the material of the Moon and terrestrial material, the conclusion was reached that the Moon was formed far away from where it is now. Thus, the logic of the hypothesis expressed by Urey is again being applied. Kaula and Harris have written ironically about this: "... The philosophy is evidently such that the prohibitions of celestial mechanics are more vulnerable than the prohibitions of thermochemistry, since in celestial mechanics one usually considers a smaller number of bodies" [78].

It is appropriate to present here the counter arguments against the capture of a finished Moon, which were expressed in work [78], since it contains a direct comparison of the effectiveness of capture by collisions with capture with the help of tidal dissipation of energy.

Let us evaluate the dissipation of energy because of tidal friction with one approach of the Moon to the Earth (such is the requirement of the hypotheses of [70, 74 - 77], the most difficult to satisfy).

For a given dissipation factor $1/Q_{\mathfrak{C}}$ and perigee r_p, the dissipation of energy with one passage on a greatly elongated orbit due to tidal friction can be easily evaluated [79], since it is consumed primarily in the radial tides in the Moon. For a homogeneous Moon moving from some distance up to perigee:

$$\Delta E = \frac{1}{Q}\int\limits_{\text{over volume}}\int\rho\dot{u}_i\frac{(U+T)}{r_i}\,dS\,dt = -\frac{2}{Q}\int\limits_{r_p}^{\infty}\int\limits_{\text{over surface}}\rho\frac{h}{g}\frac{\partial}{\partial r}(U+T)\,ds\,dr = -\frac{2}{Q}\int\limits_{r_p}^{\infty}2\pi R_{\mathfrak{C}}^2\int\limits_0^{\pi}\frac{m}{4\pi R_{\mathfrak{C}}^3/3}\frac{h}{Gm/R_{\mathfrak{C}}^2}\left[-\frac{3GMR^2}{r^4}P_{20}(\theta)\right]\times\left[\frac{(1+k)GMR_{\mathfrak{C}}^2}{r^3}P_{20}(\theta)\,d\theta\,dr = \frac{3}{10}\frac{h(1+k)R^5GM^2}{Q_{\mathfrak{C}}r_p^6}\right. \tag{3.11}$$

where h, k are the Love numbers approximately equalling 0.033 and 0.020 /90 for the present Moon [80]; $R_{\mathfrak{C}}$ is the radius of the Moon; M is the mass of the Earth; U is the gravitational potential of the Earth; T is the potential of the tidal deformation; u_1 is the radial shift of the surface of the Moon; ρ is its density; g is the gravitational acceleration at its surface. By taking $Q_{\mathfrak{C}} = 10$ and the minimum $r_p = 1.5\ R$ in accordance with work [66], we obtain from Equation (3.11) per passage:

$$\Delta E = 0{,}5\cdot 10^{33}\ \text{erg} \tag{3.12}$$

We note that ΔE is inversely proportional to the sixth power of the perigee distance r_p. Thus, the assumption of $r_p = 1.5\ R$ by Gerstenkorn leads to a maximum estimate of ΔE. We have already noted the weakness of the concept [66] in connection with the disruption of the Moon at such close distances. It is interesting to compare the obtained value of ΔE with the energy loss with collision.

For the collision of the Moon of mass m and velocity v with another body of mass m' and velocity v' directed counter to v, we have the relations:

$$\Delta E = \frac{1}{2}[mv^2 + m'v'^2 - (m+m')v_0^2], \tag{3.13}$$

$$mv - m'v' = (m+m')v_0. \tag{3.14}$$

By substituting v_0 from (3.14) into (3.13) and taking v = 10 km/sec and v' = 7 km/sec, we find the energy loss ΔE for collision, which equals $0.5\cdot 10^{33}$ ergs for $m' = 1.4\cdot 10^{21}$ gm. Thus, it is sufficient for the Moon to collide with a body of at most $2\cdot 10^{-5}$ of its mass (radius

of $\sim$ 50 km) such that this gives a more effective dissipation of energy than tidal friction.

It should be noted that for an actual capture of the finished Moon, the tidal loss of energy of $\Delta E = 0.5 \cdot 10^{33}$ ergs, found in (3.12), for the closest approach of the Moon to the Earth is not an estimate of the energy loss required for an actual capture of the finished Moon. The required loss is at least 2 orders of magnitude greater.

The minimum loss of energy of the motion of the Moon for its capture by the Earth should be understood as the loss of kinetic energy of its motion "at infinity" in the geocentric system (cf. following chapter). Only in the unique case, when the Moon is moving in almost the same orbit with the Earth and its geocentric velocity "at infinity" (v_∞) is at most 40 m/sec, would such a small energy loss ΔE with its approach toward the Earth be sufficient in order that a parabolic orbit be converted into an elliptical orbit. But motion along the same orbit is impossible without perturbations. Thus, in actuality, v_∞ is always much greater. In the hypotheses of [74 - 77], the assumption about the orbit of the Moon close to the Earth does not solve the problems, a very distant orbit is necessary at first. Thus, for example, in the hypothesis of Cameron [75] the Moon approaches from the orbit of Mercury. Then $v_\infty \approx 7.5$ km/sec, $\Delta E \approx 2 \cdot 10^{37}$ ergs, i.e. $4 \cdot 10^4$ times greater than the clearly overestimated contribution of tidal dissipation, which was evaluated in (3.12). Only a collision with a body comparable in mass could slow down the Moon for its transition to a closed geocentric orbit. And this in turn, as was already indicated above, is practically impossible.

Kaula and Harris [78] again emphasize the conclusion of Kaula [28] that the formation of a circumterrestrial swarm of bodies and particles due to inelastic collisions close to the growing Earth, in agreement with our scheme (cf. Chapters 4 and 5), remains the most acceptable model of the origin of the Moon from the dynamic point of view. Chemical differentiation of material must be investigated within the framework of a correct dynamic model. Kaula and Harris develop these opinions in their new works [81, 82].

Thus, neither the hypotheses of the splitting off of the Moon from the Earth nor the hypotheses of the capture of a finished Moon by the Earth are capable of overcoming the difficulties arising with the explanation of the physical, mechanical and chemical properties of the Moon. The only acceptable scheme is the formation of the Moon in an orbit about the Earth during the process of growth of the Earth.

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CHAPTER 4.

FORMATION OF THE CIRCUMTERRESTRIAL SATELLITE SWARM DURING THE ACCUMULATION OF THE EARTH

4.1 Growth of the Earth.

The orbits of the terrestrial planets: Mercury, Venus, the Earth and Mars - are separated from one another by very great distances, which exceed the radii of their spheres of influence by 50 - 100 times*. These distances aid the stability of their motions about the Sun [1], and also the autonomy of their satellite systems. There are no reasons to assume that the orbits of the terrestrial planets were significantly different in the past. The tidal interactions of the planets with the Sun is negligibly small, it could influence their distances only after 10^{15} - 10^{22} years [2]. The ejection of bodies from the zone of the terrestrial planets far beyond the limit of this zone is practically excluded, since none of the planets could be an effective "ejector" for objects close to them [3]. Thus, the surface density σ and dimensions of the supply zone in the preplanetary cloud are evaluated from the present masses and distances of the planets. /95

The supply zone for the Earth can be assumed a ring with an inner radius of about 0.8 AU and an outer radius of 1.3 AU. The value of σ for the Earth is about 10 gm/cm^2. The rate of growth of a nucleus of mass m is being formed within its zone in a circular orbit about the Sun is:

$$\frac{dm}{dt} = \pi l^2 \rho v. \tag{4.1}$$

Here πl^2 is the gravitational cross section of the growing body defined by the formula:

$$\pi l^2 = \pi r^2 (1 + v_e^2/v^2), \tag{4.2}$$

where r is the radius of the body; $v_e = \sqrt{2Gm/r}$ is the escape velocity at its surface; v is the average "geocentric" velocity, i.e. the velocity

* By the sphere of influence of the planets is meant that region of space in which it is suitable to take the planet as the central body in calculating perturbations, and the Sun as the perturbation. The radius of the spheres of influence of the planets is $\mathfrak{R}(m/M_\odot)^{2/5}$, where $\mathfrak{R}$ is the distance of the planet from the Sun.

of the chaotic motions of particles superimposed on their ordered motion on a Keplerian orbit with a radius of 1 AU; ρ is the spatial density of the particles being scooped up. The product ρv, in which both factors are variables, can be replaced by a more definite value [4]: /96

$$\rho v = \rho \frac{4H}{P} = \frac{4\sigma}{P} = \frac{4\sigma_0}{P}(1 - m/M). \qquad (4.3)$$

Here M is the present mass of the planet; H is the "uniform thickness" of the disc of the dust component of the cloud in the vertical direction; σ_0 is the initial surface density of the cloud, its value equalling $\sigma = \rho H$; $P = 2\pi/\sqrt{GM_\odot/\mathfrak{R}^3}$ is the period of revolution of the body about the Sun, and v and H are related by the relations:

$$H = \pi\mathfrak{R}v/2v_c, \quad v_c = \sqrt{GM_\odot/\mathfrak{R}}, \qquad (4.4)$$

which implies that the larger the velocity v of the chaotic motions of the particles, the thicker the dust disc. This velocity is determined mainly by the perturbations from the largest bodies and is written in the form:

$$v = \sqrt{Gm/\theta r}, \qquad (4.5)$$

where m and r are the mass and radius of the growing nucleus, and the numerical parameter θ is usually several times unity, varying depending on the environment in the supply zone. With the presence of gas inhibiting the motion of the particles, θ reaches several times 10; without gas and for m = 1/2 M θ $\tilde{\sim}$ 3 - 5 [4]. Substituting (4.2) - (4.5) into formula (4.1), we finally obtain:

$$\frac{dm}{dt} = \frac{4\pi\sigma r^2}{P}(1 + 2\theta) = \frac{4\pi\sigma_0(1 - m/M)}{P} r^2(1 + 2\theta). \qquad (4.6)$$

The integration of this formula leads to the dependence reproduced graphically in Figure 4.1. It is seen that 97-98% of the mass of the Earth is collected over a time of about 100 million years and that the most active stage, when the growth of mass is approximately linear with time and at the highest rate, occupies an interval of about 40 million years. The "active" stage of growth implies physically that the nucleus of the Earth had already reached a significant mass and at the same time there was still sufficient preplanetary material in the zone of the Earth. It is most natural to coordinate the formation of the circumterrestrial satellite swarm, from which the Moon formed, to this "active" stage of growth of the Earth. The following paragraphs will be devoted to a quantitative consideration of this problem.

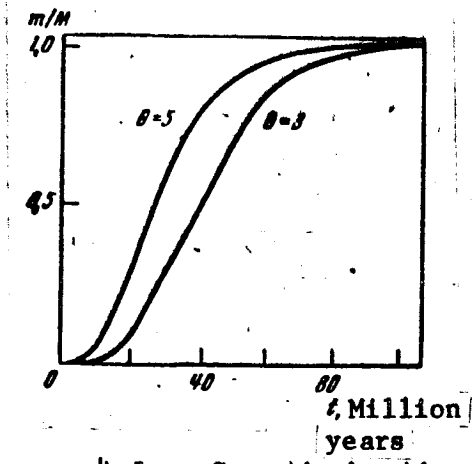

Figure 4.1. Growth in the mass of the Earth with time [4].

Let us turn again to formula (4.1). In order to increase the rate of growth of the Earth dm/dt by several orders of magnitude, either ρv or the gravitational cross section πl^2 should be increased. Both these versions are unacceptable [5-7]. An increase of ρv or, equivalently, σ_0 implies an increase in the amount of condensed material in the zone of the Earth. /97 However, if σ_0 would have exceeded the value determined by the present mass of the Earth M, then the unsolvable problem would arise: how would the surplus material in the zone of the Earth be thrown away. We have already mentioned that ejection would be practically impossible. The material would have to have be combined with the Earth, and this would have implied that the Earth be formed with a mass much greater than the present mass. It is then impossible to decrease the mass of the Earth to the required value. An increase of the gravitational cross section πl^2 as compared to the geometric πr^2 would be formally possible in the case of very low random velocities, i.e. $v/v_e \ll 1$ or, equivalently, $\theta \gg 1$. A transition from the time scale of 10^8 to 10^5 years would imply taking $\theta \approx 10^3$. However, it would seem with this that all the particles supplying the Earth would move in almost circular orbits and its nucleus, also moving in a circular orbit, would scoop up only a very narrow zone. In this case, the result of accumulation would not be the formation of planets of the type of the Earth, Venus, Mars, but the formation of a much larger number of small planets, whose orbits would be arranged much closer together than the orbits of the present planets. Thus, it is seen that it is impossible to vary arbitrarily the time of growth of the Earth.

The duration of the growth of the Earth determines the initial temperature of the Earth, i.e. the temperature from which the thermal history proper begins. The model of the growth of the Earth due to small bodies only, when the heat generated with collisions is instantaneously radiated from the surface, provides sufficient heat for melting the entire Earth only for a short scale ($\sim 10^5$ years) and even

with consideration of the energy of compression and radioactive heating in the process of growth over 10^8 years gives only about 900-1000° at the center of the Earth [8]. Consideration of the fall of large bodies to the Earth leads to a higher temperature of the outer mantle and individual centers of melting even for a scale of 10^8 years [4].

4.2. Capture of Particles from Heliocentric Orbits into the Circumterrestrial Swarm by Inelastic Collisions.

Region of existence of the satellite swarm. We take as a basis the model of growth of the Earth discussed in Chapter 3.1. Let us consider the case when the Earth, because of its direct growth, had significantly passed in mass the other particles and bodies revolving about the Sun in its supply zone, and at the same time when there is still a significant amount of material not scooped up in its zone. The capture of particles from heliocentric orbits into satellite orbits of the Earth is conveniently considered in the framework of the plane-bounded three-body problem. For this, we take the Sun and the growing Earth as the finite masses, and the particles as bodies of zero mass. We assume for simplicity that all the particles move in one plane xy. We assume that the Earth m revolves around the Sun $M_\odot$ in a circular Keplerian orbit with an angular velocity ω_c, and we take the coordinate system with origin at the center of mass of $M_\odot$ and m, rotating uniformly with the same angular velocity ω_c. Let the x axis be directed along the line of centers of $M_\odot$ and m, the y axis in the perpendicular direction in the plane of the orbit (Figure 4.2). Then, in the absence of collisions the region of motion of a small body is uniquely determined by its position (xy) and the Jacobi constant C, having the dimensions of energy per unit mass: /98

$$C = 2W - v^2,$$
$$W = \frac{\omega_c^2}{2}(x^2 + y^2) + \frac{GM_\odot}{\mathfrak{R}} + \frac{Gm}{l}. \tag{4.7}$$

Here $\mathfrak{R}$ is the distance of the particle from the Sun, l its distance from the Earth, v is the velocity of the particle in the rotating system, or the geocentric velocity.

Depending on the constant C and the coordinates of the particle, it could be a satellite of a Sun, satellite of the planet or have an

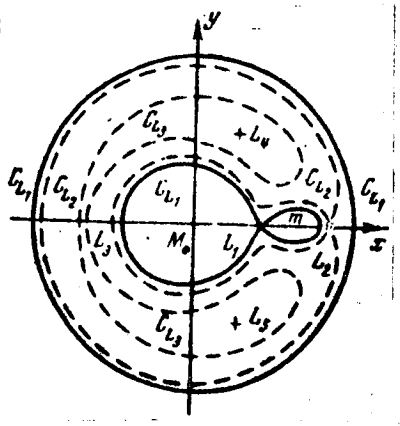

Figure 4.2. Surfaces of zero velocity in the bounded circular three-body problem.

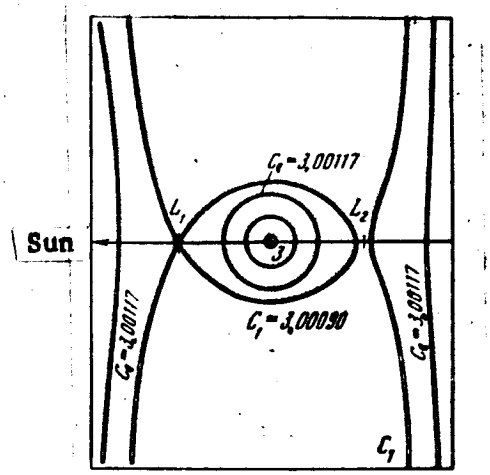

Figure 4.3. Form of the surfaces of zero velocity near the Earth.

orbit encompassing both gravitational centers. These regions are bounded by the so-called surfaces of zero velocity, which are shown in Figure 4.2, where the region surrounding the Earth is shown magnified. Under certain special conditions, other forms of motion can also be accomplished, for example periodic orbits about the libration centers - the singular points L_1 of the surfaces of zero velocity [9]. There also exist particular cases of resonance motions, when the period of the particle is in a simple ratio to the period of the planet (of the type 1: 1, 1: 2, 2: 3, 3: 4, etc). In such cases the trajectories of the particle in the rotating coordinate system usually have the form of repeating loops located between the corresponding surfaces. According to [10], all the derivations with respect to the surfaces of zero velocity, which are obtained in the bounded three-body problem for empty space, are also valid for space filled /99
with diffuse material, only if the mass of the nucleus of the Earth is greater than some limit. According to the estimate of V.V. Radziyevskiy [11], this lower limit for m is 10^{-3} M_{pres}. We will consider the growing Earth with a mass comparable with the present mass (in the interval 0.1 - 1.0 M_{pres}), which is significantly greater than the lower limit of applicability of the bounded three-body problem.

The form of the surfaces of zero velocity in the nearest vicinity of the Earth is shown in Figure 4.3. The values of the constant C are shown here for the present orbit of the Moon and for the limiting surface of zero velocity, closed with respect to the Earth, which passes

through the libration point L_1. This surface has the form of an hour glass with its neck at the point L_1. The values of C are calculated in the system of units, in which the unit of mass is the sum of the masses $M_\odot + m$, the unit of distance is the distance between the centers of $M_\odot$ and m and the unit of time is the period of one revolution of m divided by 2π. The gravitational constant then becomes equal to unity, and the linear velocity v_c of revolution of the Earth along its orbit also equals unity. The distance of the libration point L_1 from the Earth m equals:

$$l_1 = (m/3)^{1/3} - {}^1/_3(m/3)^{1/3}, \tag{4.8}$$

with an accuracy to terms of order m, and the corresponding Jacobi constant:

$$C_1 = 3 + 9(m/3)^{1/3} - \ldots \simeq 3 + 9l_1^2. \tag{4.9}$$

For the present mass of the Earth $l_1 = 0{,}01$ AU or, equivalently, 235 R. The orbit of the Moon at present stands at a distance from the Earth approximately 1/4 of the distance to l_1. It has Hill stability, i.e. in the absence of dissipative forces it could never move beyond the closed surface of zero velocity with the constant $C_1 = 3{,}00117$. One can assume in the plane case that the region of existence of satellite orbits is bounded by a circle of radius l_1, and in the three-dimensional case by a sphere with the same radius (the Hill sphere, cf. footnote on page 92). It is true that the region of existence of stable satellite orbits is somewhat smaller [12].

There is interest in calculating the values of the constants C /100 for those particles which are not gravitationally bound to the Earth and describing heliocentric orbits, which can intersect the orbit of the Earth. For eccentricities of the particle orbits $e \approx v/v_c \approx \sqrt{Gm/\theta r}$ the values of C for such particles lie approximately within the range between 3.00 and 2.80, which occupies the shaded region III in Figure 4.4. This figure also shows the distribution of the constants C for Earth satellites (I) and also for particles moving in circular orbits around the Sun (II).

Two conditions are required for capture of a particle from a heliocentric orbit into a closed geocentric orbit: passage of a particle at a distance $l \leqslant l_1$ from the Earth and a change (increase) of the constant

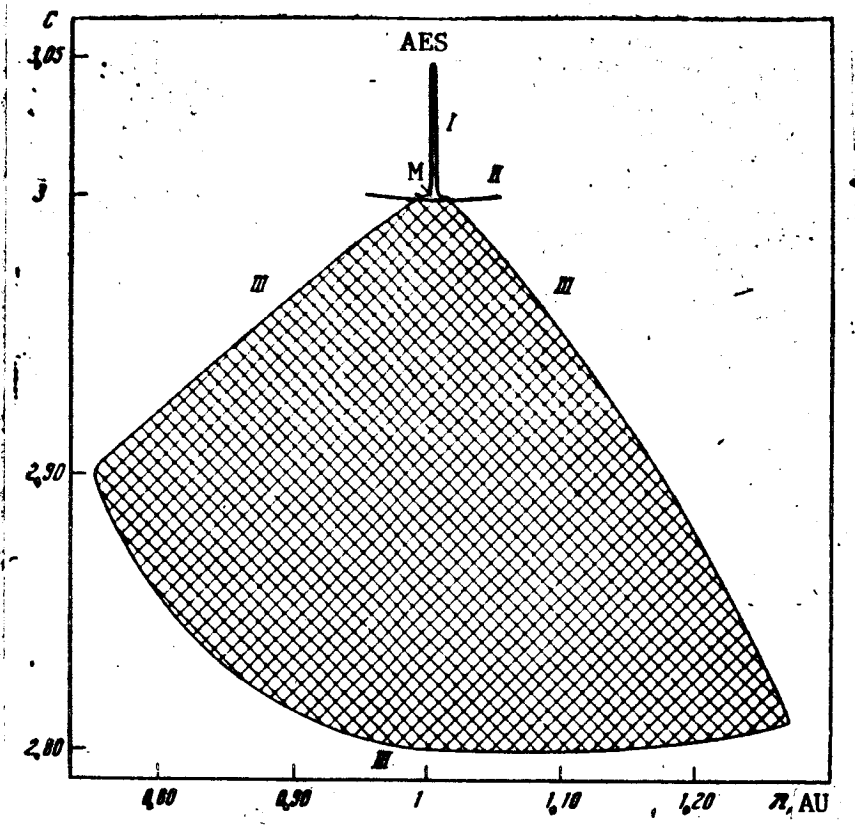

Figure 4.4. Distribution of the constants C for particles of the preplanetary cloud, which could approach the Earth at a distance less than the distance of the libration point L_1.

C to values corresponding to the values of C for satellites.

The passage of particles in the vicinity of the Earth lasts less than one year; over this time the function W does not experience significant changes due to the variability of $(x^2 + y^2)$ and m. The quantity l is variable, however the contribution of the change in the term $2Gm/l$ is cancelled with the acceleration of the particle in the gravitational field of the Earth and the increase in velocity v. The constant C can change rapidly only due to a decrease in the value of v^2, i.e. the kinetic energy of the particle in the coordinate system bound to the Earth. Such a decrease of v^2 can begin only because of a collision of the particle with one similar to itself. The increment of the constant C will be: /101

$$\Delta C = -2v\Delta v. \qquad (4.10)$$

For perfectly elastic collisions, the values of Δv equal zero on the average and $\Delta\bar{C} = 0$. However, we have the right to expect that

collisions of stony or iron-stony particles occur inelastically, where the velocity loss $\overline{\Delta v}$ has the value $\alpha\bar{v}$, where $\alpha \tilde{\sim} 1/2$.

The role of inelastic collisions in the evolution of the preplanetary material was first clearly revealed in the works of O.Yu. Shmidt [13] and L.E. Gurevich and A.I. Lebedinskiy [14]. There are much experimental data at present on the collisions of bodies with cosmic velocities of several kilometers per second, which verify the significant inelasticity of the collisions. The kinetic energy of the particles is expended in heating, melting, evaporation, fracturing and only partially into the ejection of fragments with residual velocities.

Thus, for inelastic collisions $\Delta\bar{v}<0$, $\overline{\Delta C}\simeq-\bar{v}^2$, i.e. $\overline{\Delta C}>0$, and the process of particle capture from heliocentric orbits into the circumterrestrial swarm must occur irreversibly. A numerical estimate of $\overline{\Delta C}$ can be made with the simplifying assumption that all particle velocities v are the same and equal $\sqrt{Gm/\theta r}$; then in the selected system of units for $\theta \tilde{\sim} 3$ [4] and $m = 1/2\ M_{pres}$:

$$\overline{\Delta C} \approx v^2 \approx \frac{m}{\theta r} \approx \frac{0{,}5 \cdot 3 \cdot 10^{-6}}{3 \cdot 3{,}4 \cdot 10^{-5}} \cong 1{,}5 \cdot 10^{-2}. \qquad (4.11)$$

This increment $\overline{\Delta C}$ is significantly greater than the difference between C_ζ and C_1, which is:

$$C_\zeta - C_1 = 3{,}00117 - 3{,}00090 = 2{,}7 \cdot 10^{-6}. \qquad (4.12)$$

Thus, inelastic collisions are quite able to provide for the capture of particles into satellite orbits, if these particles had constants C close to C_1 before the collisions. At the same time, particles can also approach the Earth with constants $C \tilde{\sim} 2.80$ (cf. Figure 4.4). For such particles, the increment $\overline{\Delta C} \tilde{\sim} 10^{-2}$ is insufficient, since $\Delta C \simeq 2 \cdot 10^{-1}$ is required. Thus, inelastic collisions should be considered as a mechanism leading not to a 100%-capture of all colliding particles, but only to the capture of a specific fraction of them. In connection with this, it is suitable to introduce the concept of the probability of capture for one collision. /102

Probability of capture. The distribution of the constants C in Figure 4.4 reflects the range for particles moving within the entire zone from which the Earth scooped up material. But the distribution

of the constants C for particles most frequently passing through the sphere of influence of the Earth has another form, that is a monotonic increase up to the value C = 3.0, since particles moving in orbits close to that of the Earth have the greatest probability of moving close to the Earth. With an inelastic collision accompanied by an increment $\Delta C \tilde{\sim} 10^{-2}$, such particles are captured with a probability close to unity. However, particle collisions can differ greatly in nature from each other: from head-on with oppositely directed velocities to overtaking or only lightly grazing collisions. The differences in masses of the colliding particles must also affect the velocity loss of each given particle. Consequently, the value of Δv must vary over a wide range from $\Delta v = 0$ to $\Delta v \leq v$, and ΔC varies accordingly. The variety of conditions are such here that it is difficult to describe ΔC in any sort of analytic form. Thus, we will limit ourselves to those collisions, which occur between bodies of comparable mass, and introduce the concept of the probability of capture with one collision $\bar{p}$, which is numerically equal to the average fraction of the mass of the initial colliding particles, which is captured into a circumterrestrial orbit:

$$\bar{p} = \frac{\overline{\Delta\mu}}{\mu_1 + \mu_2}, \tag{4.13}$$

where $\Delta\mu$ is the captured mass, μ_1 and μ_2 are the comparable masses of the two colliding particles; $\mu_1 \tilde{\leq} \alpha\mu_2$, where $1/2 \leq \alpha \leq 2$ [15-17].

Let us consider the dependence of the probability of capture on the distance to the Earth. The capture of particles colliding close to the limiting Hill surface is very unlikely, since it requires a zero residual velocity with respect to the Earth. Capture at the present distance of the Moon from the Earth is possible for residual velocities the order of 1 km/sec and close to the surface of the present Earth - up to 11 km/sec. For a mass of the Earth equalling half the present mass, these velocities are 0.7 and 8 km/sec respectively. One can assume that almost every particle experiencing a collision with a similar particle near the surface of the Earth will be captured into the satellite swarm (or fall to the Earth). From these considerations, the probability of capture must be close to unity at the surface of the Earth and decrease monotonically to zero at the periphery of the sphere of influence of the Earth. One can try to evaluate the form of

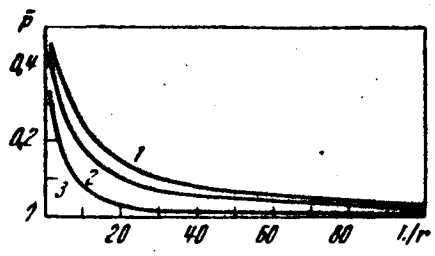

Figure 4.5. Dependence of the probability of capture of particles on the distance to the Earth for $\beta=1/2$ 1 - $\theta = 5$; 2 - $\theta = 3$; 3 - $\theta = 1$.

the dependence $\bar{p}(l)$ from the residual energy of the particles.

We assume for simplicity that all the particles and bodies have the same geocentric velocity "at infinity" $v = \sqrt{Gm/\theta r}$. In approaching the Earth to a distance of l, particles are accelerated and their geocentric velocities increase to the value: /103

$$v_1 = \sqrt{\frac{Gm}{\theta r} + \frac{2Gm}{l}}. \qquad (4.14)$$

The directions of the velocities v_1 are random, as are the directions of v, since the bodies approach the Earth from all sides. If one body collides at a distance l from the Earth with another similar to it, then both will lose on the average a certain fraction (1 - w) of their specific energy v_1^2 in the collision. These bodies or their fragments will be captured into circumterrestrial orbits, if the remaining fraction w_c of the kinetic energy of the geocentric motion of the bodies is less than the energy required for escape from the gravitational field of the Earth at the distance l, i.e.:

$$w_c v_1^2 = w_c\left(\frac{Gm}{\theta r} + \frac{2Gm}{l}\right) < \frac{2Gm}{l}. \qquad (4.15)$$

Hence, the condition for capture:

$$w_c < \frac{2\theta r}{l + 2\theta r}, \qquad (4.16)$$

i.e. the allowable fraction of the remaining energy must decrease with distance. In the range of velocities from 1 to 10 km/sec, the relative loss of energy with collisions evidently dies not depend significantly on v (at least, does not decrease). Considering that w is distributed randomly for collisions and that this distribution does not depend on l (i.e. that w does not depend on v_1), we can conclude that the condition (4.16) is more easily satisfied for small l that for large. We assume that the probability of capture is proportional to the right side of the inequality (4.16) with a coefficient $\beta<1$ and, moreover, we introduce into $\bar{p}(l)$ a factor $1 - l/l_1$, which causes $\bar{p}$ to vanish at the boundary of the swarm. We obtain:

$$\bar{p}(l) = \beta \frac{2\theta r}{l + 2\theta r}(1 - l/l_1). \qquad (4.17)$$

The form of the curve of $\bar{p}(l)$ is shown in Figure 4.5 for $\theta=1,3,5$ and $\beta=1/2$. In this case, the probability of capture falls from approximately 1/2 at the surface of the Earth to zero at the boundary of the swarm. It should be noted that, along with the capture of particles at the periphery of the swarm, ejection of particles from the sphere of influence of the Earth is also possible with the help of new collisions. /104

The concept of the probability of capture $\bar{p}(l)$ as the relative fraction of mass captured with one collision of previously free particles can also be extended to the case of collisions of a free particle with a particle already captured into the swarm. Since the input of new material here, other conditions being equal, is two times less than for "free-free" collisions, one can take:

$$\bar{p}_2(l) = {}^1/_2\ \bar{p}_1(l), \tag{4.18}$$

where $\bar{p}_1$ refers to the "free-free" collisions, and $\bar{p}_2$ to "free-bound" collisions.

Amount of captured material with "free-free" collisions. The mass of the material captured into the circumterrestrial swarm was determined by the number of mutually colliding particles and the probability of capture. Collisions occurred over an immense range of variation of sizes of the preplanetary particles and bodies - from bodies with dimensions of the largest asteroids (10^7 - 10^8 cm) to the finest dust (10^{-4} - 10^{-5} cm). We assume that their size distribution (outside the swarm) was described by a power law:

$$dN(a) = B_1 a^{-n_1} da. \tag{4.19}$$

The distribution of many ensembles existing now, for example, asteroids and meterorites, and also bodies, which at one time formed the lunar and Martian craters, can be represented in this form. Because of the random velocities, collisions of particles with any size ratio must occur. But not all, even completely inelastic collisions can lead to a sufficient retardation of the particle. For example, a body of asteroid dimensions does not lose its velocity in colliding with dust. Thus, we simply will not consider collisions of large bodies with small, since they cannot lead to capture of these bodies into the satellite swarm. As for the small particles, their collisions with large bodies lead to capture, if the large body belongs to the swarm, and to removal

from the swarm in the other case. However, collisions of small particles with large bodies occur comparatively more rarely than with small; thus, we can completely neglect the first case as compared to the second. We will take into account only those collisions which occur between particles of comparable mass, or equivalently, comparable sizes, say, for particles of radii a - only collisions with particles from a - Δa to a + Δa, where:

$$\Delta a = \alpha a, \quad \alpha < 1. \tag{4.20}$$

By assuming that the masses of particles must not differ by more than 2, we obtain $\alpha \approx 1/4 - 1/5$. The total mass Σ_1 of all particles and bodies experiencing such collisions per unit volume per unit time is determined by the integral: /105

$$\Sigma_1 = \int_{a_{max}}^{a_{min}} da \int_{a-\Delta a}^{a+\Delta a} \frac{4}{3}\pi\delta_s r^3 \pi r^2 \bar{v} \frac{1}{2}(B_1 r^{-n_1})^2 dr, \tag{4.21}$$

where δ_s is the density of particles. The constant B_1 is determined from the normalization condition:

$$\rho_1 = \int_{a_{min}}^{a_{max}} B_1 r^{-n_1} r^3 \frac{4}{3}\pi\delta_s dr = \frac{B_1}{4-n_1}\frac{4}{3}\pi\delta_s (a_{max}^{4-n_1} - a_{min}^{4-n_1}). \tag{4.22}$$

Here ρ_1 is the spatial density of material in the supply zone of the Earth. Hence:

$$B_1 = \frac{3\rho_1 (4-n_1)}{4\pi s (a_{max}^{4-n_1} - a_{min}^{4-n_1})}. \tag{4.23}$$

By integrating (4.21) with consideration of (4.23), we find:

$$\Sigma_1 = \frac{3}{4}\frac{\bar{v}\rho_1^2}{\delta}\frac{\alpha(4-n_1)^2}{(7-2n_1)}\frac{a_{max}^{7-2n_1} - a_{min}^{7-2n_1}}{(a_{max}^{4-n_1} - a_{min}^{4-n_1})^2} \tag{4.24}$$

(For $n \neq 3.5$ and 4.0). For $n_1 = 3.5$:

$$\Sigma_1 = \frac{3}{4}\frac{\bar{v}\rho_1^2}{\delta_s}\frac{\alpha(4-n_1)^2 \ln(a_{max}/a_{min})}{(a_{max}^{4-n_1} - a_{min}^{4-n_1})^2}, \tag{4.25}$$

and for $n_1 = 4$:

$$\Sigma_1 = \frac{3}{4}\frac{\bar{v}\rho_1^2}{\delta_s}\frac{\alpha}{(7-2n_1)}\frac{a_{max}^{7-2n_1} - a_{min}^{7-2n_1}}{\ln(a_{max}/a_{min})}. \tag{4.26}$$

We denote by a_{eff} the factor with dimensions of length, which depends on the function of the size distribution of particles:

$$a_{eff} = \frac{7-2n_1}{(4-n_1)^2}\frac{(a_{max}^{4-n_1} - a_{min}^{4-n_1})^2}{(a_{max}^{7-2n_1} - a_{min}^{7-2n_1})} \tag{4.27}$$

[In the cases $n_1 = 3.5$ or 4.0, logarithms appear in this expression; cf. (4.25) and (4.26)]. Since $a_{max} \gg a_{min}$, a_{eff} can be expressed to

a good approximation in the form:

n_1 ...	3,0	3,5	3,75	4,0	4,25
a_{eff} ...	a_{max}	$\frac{4a_{max}}{\ln(a_{max}/a_{min})}$	$8\sqrt{\frac{a_{max}}{a_{min}}}$	$a_{min}\ln^2\frac{a_{max}}{a_{min}}$	$24a_{min}$

In general, for $n_1 \leq 3.5$ the quantity a_{eff} is determined by the maximum size of the particles, and for $n_1 \geq 4.0$ by the minimum, in the interval $3.5 < n_1 < 4.0$ by a combination of the form $a_{max}^{z} a_{min}^{1-z}$, where $z < 1$. The value of a_{eff} is an important characteristic, since it determines the total mass of material of the particles experiencing collisions with similar particles: /106

$$\Sigma_1 = \frac{3}{4}\frac{\bar{v}p_1^2\alpha}{\delta_s a_{eff}}. \tag{4.28}$$

The flux of mass of material μ_1 captured into the swarm equals the mass Σ_1 multiplied by the probability of capture $\bar{p}_1$:

$$\frac{d\mu_1}{dt} = \bar{p}_1\Sigma_1. \tag{4.29}$$

Formula (4.29) gives us the flux of material into the satellite swarm per unit volume per unit time due to collisions of previously free particles. The same expression is obtained, if one considers not only collisions in the interval of comparability a + Δa, but also all other collisions, by setting their probability of capture equal to zero.

Change of the particle size distribution function with their capture into the swarm. Let us consider how the particle size distribution law changes with their entry into the swarm. We first assume that there is no fractionation of particles, then the number of particles $dN_2(a)$ with radii in the interval from a to a + da will be supplied only by their collisions with particles of radii from a to a + Δa:

$$dN_2(a) = {}^1/_2 v\pi\,(2a)^2\Delta N_1(a)\,dN_1(a). \tag{4.30}$$

Substituting the expression for $dN_1(a)$ from (4.16), we obtain:

$$dN_2(a) = {}^1/_2 v\pi\,(2a)^2 B_1^2 a^{-2n_1}\Delta a\,da. \tag{4.31}$$

Since $\Delta a \sim a$, we have the new size distribution in the form:

$$dN_2(a) = B_2 a^{-n_2}da, \tag{4.32}$$

where B_2 does not depend on a, and the new exponent is:

$$n_2 = 2n_1 - 3. \tag{4.33}$$

Consequently, capture into the swarm due to "free-free" collisions leads to an increase of the exponent of the inverse power law (4.19) for $n_1 > 3$. The values of n_1 transform into the following values of n_2:

n_1	2	2,5	3	3,5	3,75
n_2	1	2	3	4,0	4,5

The distribution of the form (4.19) with an exponent $n_1 = 3.5$ [4,18] was evidently the most typical at the stage of preplanetary bodies. The study of the asymptotic behavior of large ensembles of solid bodies with consideration of their coagulation and fractionation leads to this. For such a distribution, the main mass is concentrated in the large bodies, and the total surface area is greater for the small fraction. For $n_1 = 3.5$, the change caused by capture without fractionation leads to a distribution with an exponent $n_2 = 4$. This implies a selection in the swarm of the small component of interplanetary particles and the subsequent relative increase of the total surface area of the small fraction. This selection has practically no effect on the distribution of particles over the entire zone of the Earth (on the value of n_1), if the mass of the circumterrestrial swarm is small as compared to the mass of all the particles. The inevitable fractionation accompanying collisions additionally increases the amount of small particles. /107

Capture of material into the circumterrestrial swarm with "free-bound" collisions of particles. With the formation of the satellite swarm due to "free-free" collisions, collisions of particles flying into the swarm with particles of the swarm must occur more and more frequently, which also leads to the capture of previously free particles. One can find by analogy to (4.29) that the mass captured per unit time $d\mu_2/dt$ is

$$\frac{d\mu_2}{dt} = \bar{\rho}_2\Sigma_2 = \frac{\bar{\rho}_1}{2}\Sigma_2 = \frac{3}{4}\frac{\bar{\rho}_1 a \bar{v} \rho_1 \rho_2}{\delta_s a'_{\text{eff}}}, \tag{4.34}$$

where ρ_2 is the spatial density of the swarm, while a_{eff} is the effective dimension for the interaction of the two ensembles of particles: "free" with the distribution (4.19) and "bound" with a distribution of the same form, but, generally speaking, with another exponent n_2:

$$a'_{\text{eff}} = \frac{7 - n_1 - n_2}{(4 - n_1)(4 - n_2)} \frac{(a_{\max}^{4-n_1} - a_{\min}^{4-n_1})(a_{\max}^{4-n_2} - a_{\min}^{4-n_2})}{a_{\max}^{7-n_1-n_2} - a_{\min}^{7-n_1-n_2}}. \tag{4.35}$$

If it is assumed that n_1 during the course of the active stage of growth of the Earth and the enrichment of the circumterrestrial swarm is ap-

proximately constant and equal to 3.5, then for various n_2, which could occur in the swarm, the effective size will be expressed to a sufficient approximation in the following form:

n_2	3,5	3,75		
a'_{eff} . . .	$\frac{4a_{max}}{\ln(a_{max}/a_{min})}$	$2a_{max}^{3/4}a_{min}^{1/4}$		
n_2	4,0		4,5	5,0
a'_{eff} . . .	$\ln(a_{max}/a_{min})\cdot\sqrt{a_{max}a_{min}}$		$4\sqrt{a_{max}a_{max}}$	$3\sqrt{a_{max}a_{min}}$.

We see that in all cases for $n_2 \geq 4.0$ the effective dimension is expressed as the geometric mean between a_{max} and a_{min} and for various n_2 differs only by a small numerical factor. The difference between a_{max} and a_{min} probably reached 12 - 13 orders of magnitude. Thus, a change in the effective dimension from a value the order of a_{max} to a value on the order of $\sqrt{a_{max}\ a_{min}}$ denotes an increase of the captured mass by 6 orders of magnitude. Thus, the free-bound inelastic collisions or, equivalently, the sticking of free particles into the swarm is the most important source of enrichment of the swarm. This mechanism will be considered in more detail in the subsequent paragraphs of this chapter. We will discuss other forms of capture of material into the circumterrestrial swarm. /108

4.3. Gravitational Capture of Particles into Geocentric Orbits

The first attempts to use the theory of O.Yu. Shmidt on the accumulation of planets to the formation of satellites around them were made for purely gravitational, "collisionless" capture. It will be shown later that the "collisionless" capture has only supplementary value for the growth of the Moon. However, the theory itself is of interest, since the role of the libration points in the bounded three-body problem as peculiar "admission points" for particles transferring from heliocentric orbits into geocentric and conversely is clearly shown.

We will discuss the theory of gravitational capture of material by the growing Earth, following the work of V.V. Rasziyevskiy [11]. Let us consider a planet with a variable mass forming along a circular orbit with the center at the common center of mass of the system; the Sun surrounded by a gravitational medium, whose density changes with

distance according to a law related to a law of planetary distances, and a particle on which acts the attraction from the Sun, medium and planets and radiation repulsion from the Sun. We introduce the distance $\mathfrak{R}_0$ between the particle and the center of gravity of the system. Then in a system of units where $\omega_c = 1$ and $G = 1$, the Jacobi integral can be written in the form:

$$v^2 = \mathfrak{R}_0^2 + \frac{2(1-m)}{\mathfrak{R}} + \frac{2m}{l} - \int_1^{\mathfrak{R}} \left(\frac{\Delta m}{\mathfrak{R}^2}\right) d\mathfrak{R} - C. \tag{4.36}$$

Here m is the current mass of the Earth together with the mass of the medium included within the limiting Hill surface surrounding the Earth (it will subsequently be called the mass of the Earth); (1 - m) is the sum of the mass of the Sun and the medium included within the heliocentric sphere of radius equalling unity, which is taken as the distance between the Earth and the Sun; Δm is the mass included between the two spherical shells with radii $\mathfrak{R}$ and 1; $\Delta m > 0$ for $\mathfrak{R} > 1$; $\Delta m = 0$ for $\mathfrak{R} \leqslant 1$. For v = 0 the Jacobi integral (4.36) gives the equation of the surface of zero velocity. As has already been mentioned in section 4.2 of this chapter, according to [10], the presence of the gravitational medium does not change qualitatively the form of the surface of zero velocity and its double points L_1 and L_2, which correspond to the three-body problem in a vacuum. The distance from the Earth to the point L_1 in the presence of a medium, as in the vacuum problem, is expressed /109 in the form (4.8), only if the nucleus of the Earth reaches a mass $m > 10^{-3} M_{pres}$. The value of the constant C_1 (for the point L_1) also preserves its classical expression with an accuracy to terms of second order under the same condition about the lower limit of m (cf. (4.9)).

Thus, even at the very early stages of the growth of the Earth, the presence of a gravitational medium cannot be taken into account in evaluating l_1 and C_1. Consideration of the radiation pressure in the bounded three-body problem performed in [19] showed that the surfaces of zero velocity are significantly more deformed for particles with cross section the order of 1 cm and smaller than in the classical case of the problem. In particular, the ovaloid encompassing the Earth and passing through the point L_1 opens in the region of L_2, merging with the external Hill "screen" (cf. Figure 4.3). However, this result is valid only for a transparent preplanetary cloud, in which the Poynting-Robertson effect "sweeps out" the small particles toward the

Sun. Thus, deformation of the surface of zero velocity because of radiation pressure will not be taken into account.

We now consider the conditions for the gravitational capture of particles by the Earth caused by the variability of the mass m. In each brief interval of time, during which there is no practical change in the mass of the Earth, the problem can be considered in its classical formulation and the expression for l_1 and C_1 can be considered invariants.

Since the instantaneous velocity and the position of the particle cannot change with each instantaneous increase in mass m, then the change in m causes a corresponding change of the constant C characteristic of the given particle. As is seen from (4.36):

$$dC = -\frac{2dm}{\Re} + \frac{2dm}{l} - 2d\left(\int_1^{\Re} \frac{\Delta m}{\Re^2} d\Re\right). \tag{4.37}$$

With an increase in the mass of the Earth by dm, the mass Δm decreases accordingly, and one can assume:*

$$2d\left(\int_1^{\Re} \frac{\Delta m}{\Re^2} d\Re\right) = \frac{2dm}{\Re}$$

and, consequently:

$$dC = \left(\frac{2}{l} - \frac{4}{\Re}\right) dm. \tag{4.38}$$

On the other hand, by differentiating (4.9), we find:

$$dC_1 = \frac{2}{(m/3)^{1/3}} dm = \frac{2}{l_1} dm. \tag{4.39}$$

For particles located within the limiting closed Hill ovaloid surrounding the Sun, $l > l_1$ and, consequently, as is seen from (4.38) and (4.39): /110

$$\frac{dC}{dm} < \frac{dC_1}{dm}. \tag{4.40}$$

The growth in mass of the Earth for such particles does not lead to their gravitational capture.

Conversely, for particles already located in the ovaloid surrounding the Earth, $\Re > l$ and $l < l_1$. Thus, for them:

$$\frac{dC}{dm} > \frac{dC_1}{dm}, \tag{4.41}$$

* V. Radziyevskiy [11] assumed $\frac{dm}{\Re}$ on the right side.

from which it follows that the escape of such particles from this ovaloid is made more difficult with the growth inmass of the Earth, and is impossible in general for particles penetrating through the point L and having $C = C_1$ at the time of transition. Such particles become stable (in the Hill sense) satellites of the planet. We assume that a particle up to capture is at the point L_1, i.e. in the coordinate system fixed to the Sun; it moves in the plane of the ecliptic revolving in the forward direction both around the Sun and around the Earth with the same sidereal periods equalling one year. After capture such a particle will then revolve around the Earth in the plane of the ecliptic in the forward direction, while its specific angular momentum with respect to the Earth will equal the momentum of the libration point L_1, i.e.:

$$k = \omega_c l_1^2 = \omega_c (m/3)^{2/3}. \qquad (4.42)$$

For the present mass of the Earth, this specific momentum would correspond to the geocentric orbit of a satellite with radius of about 78 R, i.e. even somewhat greater than the present distance of the Moon from the Earth. V.V. Radziyevskiy proves that close to the libration points L_1 and L_2, accumulations of particles of the preplanetary cloud could form, which perform oscillatory motions with respect to the centers L_1 and L_2, from which material was also scooped up in the circumterrestrial swarm with increasing mass of the Earth.

This theory was applied by Ye.P. Razbitnaya [20] to the problem of the origin of the Moon. Gravitational capture of particles was thought to be the main supply source for the Moon, but in order to obtain the Moon with its present orbital momentum, a small contribution of particles striking the Moon directly and carrying zero momentum on the average was assumed. It was assumed that the particles captured by the Earth do not stay for an indefinitely long time close to the points L_1 and L_2, but leave there because of the collision interaction in the cloud. Thus, collisions of particles played a supplementary role, redistributing the constant C. The number of particles with constants C close to C_1 was assumed proportional to the total amount of material in the zone of the Earth, i.e. $M_{pres} - m$, while the proportionality constant was taken from the condition that at the end of the growth of the Earth ($m = M_{pres}$) the Moon must have grown in its present orbit. /111

This led to a rather strange result: the initial mass of the Earth, for which the capture of lunar material begins, was obtained as 0.96 M_{pres}, and the age of the Moon as 0.84 of the age of the Earth.

A detailed critical analysis of the model of Ye.P. Razbitnaya was given by us in 1960 [15]. All the arguments lead in essence to the fact that purely gravitational capture of material due to the increase of m is a much less effective method for enriching the swarm than inelastic collisions. Actually, we compare both mechanisms in terms of the increment of the constant C for one approach of a particle to the Earth. The flight of the particle inside the sphere of influence of the Earth lasts less than one year (the period of revolution of the point L_1). Over this time the growth of the mass of the Earth gives:

$$\Delta C = 2\,\Delta m/l \qquad (4.43)$$

independently of the size of the particle. We take the minimum value for l equalling r and the time of flight equalling one year to maximize the right side. Then in the system of units taken in the bounded three-body problem:

$$\Delta C \ll 2\,\frac{(\Delta m/m)_1\,\mathrm{yr}\,[m/(M_\odot+m)]}{r/\mathfrak{R}} = \frac{2\cdot 10^{-6}\cdot 3\cdot 10^{-6}}{4\cdot 10^{-3}} \approx 1{,}5\cdot 10^{-9}. \qquad (4.44)$$

Actual ΔC will be less by approximately 10^2 times, since the close flight of a particle at distances $l \approx r$ lasts hours, while the main time the order of a year is spent in distant flight at $l \approx 100\,r$. The rate of growth Δm was also taken as a maximum here; it corresponds to the active stage of grwoth of the Earth [4], and not to the stage of attenuation of the growth from 0.96 to 1.00 M_{pres}, as by Ye.P. Razbitnaya. Only a completely negligible fraction of all the particles filling the space in the zone of the Earth can be captured with such a small ($\tilde{\sim} 10^{-11}$) increment of ΔC. We have already noted above that capture usually requires ΔC from 10^{-1} to 10^{-4}.

An estimate of ΔC for inelastic collisions depends strongly on the particle size and on the size distribution function for the particles of the swarm. By using the equations presented below in the paragraph on the relaxation time of the swarm, one can find ΔC for various conditions of the swarm. Here $\Delta C \tilde{\sim} 2v\Delta v$, while $\Delta v \tilde{\sim} 1/2$ for head-on collisions of comparable masses. In the units being used, this corresponds to $\Delta C \tilde{\sim} 10^{-2}$ (cf. (4.11)). By taking into account

that not all collisions can give a large velocity loss, we take for comparable masses $\overline{\Delta C} \approx 10^{-3}$. For the collision of a particle of m_1 with a smaller mass m_2, we will then have $\Delta C \approx 10^{-3} \cdot m_2/m_1$. Even if there is not yet a swarm in the present sense, and the sphere of influence of the Earth is simply filled by "background" particles, i.e. the interplanetary medium with density the order of 10^{-12} gm/cm^3, which is typical for the active stage of growth of the Earth ($m \approx 1/2\ M_{pres}$), then about $2 \cdot 10^{-4}$ the mass of the Moon must be contained /112 within the sphere of influence of the Earth (we have neglected here the focussing of particles close to the Earth, which also increases the total mass of the "background" particles). One flight of a particle through the sphere of influence of the Earth filled with such a medium leads a particle of radius 10^{-3} cm to a collision with a similar particle (or with a sum of smaller particles having the same mass), a particle with radius 10 cm encounters 10^{-2}, a body of radius 1 km 10^{-4} and 100 km $\sim 10^{-5}$ of its mass. The increment ΔC is $\sim 10^{-3}$, 10^{-5}, 10^{-7}, 10^{-8} respectively. We see that only one deceleration in the medium surrounding the Earth at the beginning of its growth more effectively changes the nature of the motion of particles and even large bodies than the growth of the mass of the Earth over the time of approach. The contrast increases when a swarm of satellites with a mass, say 10^{-2} the mass of the Moon already exists around the Earth. If the particle size distribution of the swarm has the form a^{-4}, then all particles with radius less than 10 m for one flight will collide with particles, whose mass exceeds its own by many times. We can take $\Delta C \geq 10^{-3}$ for them. For bodies with radius 100 km ΔC decreases to 10^{-6}. The presented comparison shows that inelastic collisions of particles should be considered the main factor in forming the circumterrestrial swarm, while purely gravitational capture is a very small supplementary supply source.

4.4. Mass of the Circumterrestrial Swarm.

We will distinguish the current mass of the circumterrestrial swarm μ_t and the total mass of material, which passed through the stage of the swarm and later combined with the Earth and with the Moon:

$$\mu_\Sigma = \int_{t_0}^{t_{pres}} \frac{d\mu}{dt}\, dt.$$

According to (4.29) and (4.34), one can write:

$$\frac{d\mu}{dt} = \int_{l_0}^{l_1} 4\pi l^2 \left[p_1(l)\,\Sigma_1 + \frac{p_1(l)}{2}\,\Sigma_2 \right] dl. \qquad (4.45)$$

Then:

$$\mu_\Sigma = \int_{t_0}^{t_{pres}} \int_{r(t)}^{l_1(t)} 4\pi l^2(t) \left[p_1(l)\,\Sigma_1(t) + \frac{p_1(l)}{2}\,\Sigma_2(t) \right] dl\,dt. \qquad (4.46)$$

Here the term with Σ_1 determines the contribution of the "free-free" collisions, and the term with Σ_2 the contribution of the "free-bound" collisions. It has already been noted that Σ_2 can give the most significant flux of material into the swarm. However, the determination of μ_Σ from Formula (4.46) is difficult just because of the inhomogeneity of the term with Σ_2. With the absence of scooping up of material of the swarm, its density ρ_2 due to "free-bound" collisions increases exponentially, since $d\rho_2 \sim \rho_2$. However, as we will see below, the densest inner part of the swarm falls out to the Earth, while the peripheral part is strongly reorganized in view of the redistribution of momentum (cf. section 4.5 of this chapter). Moreover, the growth of satellites must occur in the densest parts of the swarm. We will show later that the satellites clear the swarm more rapidly than the planet itself scoops up its zone. Thus, the current mass of the swarm μ_t, which determines its current density ρ_2, is maintained at some level over the time of active growth of the Earth, rather than experiencing an exponential growth. We will try to evaluate this "level" below, although not very definitely. /113

The beginning of the formation of the swarm and the maximum rate of its enrichment. It is difficult to indicate the time for the beginning of the formation of the swarm. It could arise only around a rather large nucleus of the Earth, already having passed its "competitors." Starting with the fact that the planet Mars has two small satellites, one can assume that for a mass of the nucleus of the Earth equalling the mass of Mars, i.e. about 0.1 M_{pres}, the formation of its satellite swarm would have already begun. However, the much earlier protions of the swarm with the subsequent growth of the Earth must have approached it in accordance with the invariant ml_s = const, where l_s is the orbital radius of the satellite. An increase of m by 10 times implies a contraction of the orbit by 10 times, and those particles, which first revolved at a distance less than 30 R, are finally within

the Roche zone (3 R) or have fallen to the Earth. Thus, only the portion of the swarm captured when the mass of the planet was comparable to the present mass is important for the accumulation of satellites. In certain special cases, one can evaluate the mass of the growing planet, when the rate of enrichment of the swarm is maximum. We introduce under the integral in (4.45) some average probability of capture $\bar{p}$ in place of $p_1(l)$, and a density of the swarm $\bar{\rho}_2$ averaged over the volume. Then $d\mu/dt$ can be written in the form:

$$\frac{d\mu}{dt} = \left(\frac{3}{4}\frac{\bar{p}\bar{v}\rho_1^2\alpha}{\delta_s a_{eff}} + \frac{3}{4}\frac{\bar{p}\bar{v}\rho_1\bar{\rho}_2\alpha}{\delta_s a'_{eff}}\right)\cdot {}^4/_3\pi l_1^3 = \frac{\pi\bar{p}l_1^3\bar{v}\alpha\rho_1}{\delta_s}\left(\frac{\rho_1}{a_{eff}} + \frac{\bar{\rho}_2}{a'_{eff}}\right). \quad (4.47)$$

We now impose the condition on $\bar{\rho}_2$ considering it always to be proportional to ρ_1. Keeping in mind that $l_1^3 \sim m$, $\bar{v} = \sqrt{Gm/\theta r} \sim m^{1/3}$ and that $\rho_1 \sim (M_{pres} - m)$, if $\mu << m$, and differentiating the right side as a function of m with respect to time, one can find for which m^* the rate of enrichment of the swarm $d\mu/dt$ is maximum, i.e. $d^2\mu/dt^2 = 0$. We have:

$$\frac{d^2\mu}{dt^2} \sim {}^4/_3 m^{1/3}\frac{dm}{dt}(M_{pres} - m)^2 + m^{4/3}[-2(M_{pres} - m)]\frac{dm}{dt} = 0. \quad (4.48)$$

Dividing the right side by $m^{1/3}dm/dt \neq 0$, we obtain an algebraic quadratic equation for m: /114

$$m^2 - {}^7/_5 M_{pres} m + {}^2/_5 M_{pres}^2 = 0; \quad (4.49)$$

Its solution is $m_1^* = 2/5\ M_{pres}$ and $m_2^* = M_{pres}$. It is obvious that the second root corresponds to the minimum of $d\mu/dt$, when this quantity vanishes at the end of the growth of the Earth. The first root $m_1^* = 2/5\ M_{pres}$ corresponds to the maximum of $d\mu/dt$. This means that the swarm grows most rapidly somewhere in the interval of values of m between 1/3 and 1.2 M_{pres}. The Earth itself grows most rapidly in this interval (cf. Figure 4.1). We have already called this stage the active stage of growth.

Dependence of the mass of the satellite swarm on the mass of the planet. One can now try to find the dependence of the total mass of the swarm on the final mass of the planet: $\mu_\Sigma(M_{pres})$. We first assume that the contribution of the material of the swarm to the mass of the entire planet can be neglected. Then, one can divide the rate of growth $d\mu/dt$ given by (4.45) into the rate of growth of the mass of the Earth from (4.6). We obtain:

$$d\mu/dm = D_1(M_{pres} - m - \mu) + D_2 m^{-1/3}\mu, \quad (4.50)$$

where the first term on the right describes the contribution of "free-

free" collisions, and the second term the "free-bound" collisions. Their coefficients are respectively:

$$D_1 = \frac{\rho \sqrt{6\alpha}}{9a_{\text{eff}}(1+2\theta)\sqrt{3\pi\delta_0 M_\odot \bar{R}}}; \tag{4.51}$$

$$D_2 = \frac{3^{1/6}\rho_2 \delta_\odot^{2/3}}{(4\pi)^{1/6}\delta_0 a_{\text{eff}}(1+2\theta)}. \tag{4.52}$$

We first evaluate only the contribution of the "free-free" collisions. We have:

$$\frac{d\mu_1}{dm} = D_1(M - m - \mu_\Sigma). \tag{4.53}$$

Neglecting μ_Σ as compared to M and m, we obtain:

$$\mu_1 = D_1\left(\frac{M_{\text{pres}}^2}{2} - M_{\text{pres}} m_0 + \frac{m_0^2}{2}\right). \tag{4.54}$$

The maximum value of μ_1 for $m_0 = 0$ will be:

$$\mu_1 = D_1 M_{\text{pres}}^2/2. \tag{4.55}$$

We evaluate this expression for $n_1 = 3.5$, $a_{\max} \approx 10^8$ cm, $a_{\min} \approx 10^{-4} \div 10^{-5}$ cm, which gives:

$$a_{\text{eff}} \approx 10^7 \text{ cm}. \tag{4.56}$$

We obtain: /115

$$\mu_1(M_{\text{pres}}) = 10^{-5}\mu_\oplus, \tag{4.57}$$

which is neglibibly small as compared to the mass of the Moon. We can conclude that only "free-free" collisions alone would not be able to form the circumterrestrial swarm required for the formation of the Moon in it.

On this basis, we will neglect the term with D_1 in Equation (4.50) and consider the formation of the circumterrestrial swarm due only to "free-bound" collisions, which gives:

$$\mu_2 = \mu_0 e^{\gamma t}. \tag{4.58}$$

The mass of the "background" particles filling the sphere of influence of the Earth can be taken for μ_0, since they fill the role of the resisting medium at first. As soon as a comparable mass of particles appears in the swarm in geocentric orbits, the role of the resisting medium passes to them and the new portions of material are then captured in "free-bound" collisions.

We assume that:

$$\mu = \mu_2, \tag{4.59}$$

$$\mu_1 \approx \gamma\mu, \tag{4.60}$$

where γ is the fraction of the total mass of the swarm at a given moment in the circumterrestrial space, $\gamma<1$, then:

$$\mu = \mu_0 e^{FT}, \tag{4.61}$$

where:

$$F = \frac{3^{1/3}\delta_\oplus^{2/3}\gamma\beta\alpha\,(m^{1/3} - m_0^{1/3})}{(4\pi)^{1/3}\delta_s\,(1+2\theta)\,a'_{\text{eff}}}. \tag{4.62}$$

The obvious condition for the formation of a massive swarm around the growing planet is $F>1$.

Taking the mass of background particles during the active stage of growth of the Earth ($m = (2/5 - 1/2)\,M_{\text{pres}}$) for μ_0, we have within the sphere of influence of the Earth:

$$\mu_0 = (10^{-4} \div 10^{-5})\,\mu_\oplus. \tag{4.63}$$

Consequently, $F \geqslant 10$ is necessary to obtain a swarm with a total mass of $10\mu_\oplus$. We set $\alpha = 1/5$, $\bar{p} = 1/100$; $m_0 = 2/5\,M_{\text{pres}}$. Then for various γ, we have the condition for a_{eff}, which gives the required value of F:

γ	1	0,1	0,1	0,001
a'_{eff}, cm .	10^3	10^4	10^3	10^3

An analogous result is obtained if one assumes that the density /116 of the material of the swarm ρ_0 (excluding large satellites) is always proportional to ρ_1, i.e. the background density, and the proportionality constant between them is k_2 ($\rho_2 = k_2\rho_1$). Then, in place of (4.34), we have:

$$\frac{d\mu_2}{dt} = \frac{\beta}{2}\Sigma'_2 = \frac{3\beta\alpha\bar{r}k_2\rho_1^2}{4\delta_s a'_{\text{eff}}}, \tag{4.64}$$

$$\mu(M) \approx D'_1\frac{M^2}{2}, \tag{4.65}$$

$$D'_1 = \frac{2p\alpha\sqrt{6}k_1k_2}{9a'_{\text{eff}}(1+2\theta)\sqrt{3\pi\delta_s M_\odot}}. \tag{4.66}$$

We introduce with ρ_1 the factor $k_1<1$ in order to take into account only those collisions of the ensemble of interplanetary particles with the ensemble of the swarm, which occur between particles of comparable mass. Excluding from the interaction, for example, the largest preplanetary bodies with radii the order of 10^8 cm (they have no "equals" in the swarm), we must set $k_1 = 1/3$ if $n_1 = 3.5$ [cf. (4.19)]. In this case, $a_{\max} \approx 10^7$ cm. For $a_{\min} = 10^{-4} - 10^{-5}$ cm and $n_2 = 4.0$, we obtain $a_{\text{eff}} = 200 - 600$ cm (cf. derivations on page 123). Substituting $\mu = 10\,\mu_\oplus$, into (4.65), we find $k_2 = 10$.

Consequently, the condition:

$$\rho_2 \approx 10\,\rho_1, \tag{4.67}$$

must be satisfied for the capture into the swarm of 10 times the mass of the Moon, i.e. if we assumed that the density of the swarm is always maintained at a specific level, then this level must be an order of magnitude greater than the "background" density ρ_1 during the active stage of grwoth of the Earth. Numerically, $\rho_1 \approx 10^{-12}$ gm/cm^3 at this stage. Then, within the present lunar orbit of 60 R, condition (4.67) gives the current mass of the swarm of about 10^{-4} the mass of the Moon, and within the sphere of influence of the Earth $(10^{-3} - 10^{-2})\mu_C$. During the evolution of the swarm, it must have been subjected to continuous renewal due to the flux of interplanetary particles and the scooping up of particles by the growing satellites of the Earth.

4.5. Geocentric Angular Momentum of the Circumterrestrial Swarm and the Initial Distance of the Moon (or System of Protomoons) from the Earth.

The planet-centered angular momentum of that portion of the material of the swarm, which combines into a satellite, determines the orbital momentum of this satellite, i.e., its initial distance. In the case when a system of satellites is formed, their total momentum is determined. The portion of the material of the swarm, which does not appear in the satellites, but falls to the planet, takes away from it some angular momentum, which is combined with the momentum being brought into the direct growth of the planet. Thus, an estimate of the planet-centered momentum of the swarm is of great interest [21]. The rotation of planets of the required order was already simulated with computers in 1965 [22,23]. Calculation of the rotation of the satellite swarm is significantly more difficult. /117

It is well known that if one takes a unit cell in the form of a circle in the plane circumsolar cloud, each particle of which describes a circular Keplerian orbit, then its angular momentum with respect to the axis passing through its geometric center and parallel to the axis of rotation of the cloud is + 1/4 $I\omega_c$, where the plus sign denotes rotation in the same direction as for the cloud with respect to the Sun. The quantity I denotes the moment of inertia of the cell with respect

to this axis, while ω_c is the Keplerian angular velocity about the Sun. The presence of a nucleus of a planet at the center of the cell must somehow change the value of its planet-centered momentum.

Let us consider the Earth-swarm system at the stage of formation. Just the captured particles can have orbits with various eccentricities and various directions of revolution about the Earth. Subsequent inelastic collisions with other particles of the swarm shift them to almost circular orbits in the central plane of the swarm or lead to their fall to the Earth. If the random components of the residual velocities of just the captured particles are symmetric with respect to the system of coordinates rotating along with the Earth about the Sun (as was assumed in the works of V.V. Radziyevskiy and A.V. Artem'yev [24,25]), then the total angular momentum of the swarm is $+I\omega_c$, where I is the moment of inertia of the particles of the swarm during the time of capture with respect to an axis passing through the Earth and parallel to the axis of the ecliptic. The conglomerate of particles stably bound to the Hill sphere would have such an angular momentum. Thus, one sometimes speaks of the solid-body rotation of the new portion of the swarm with the velocity of revolution of the Earth about the Sun.

Captures per unit volume are accomplished more frequently close to the Earth, but because of the great elongation of the orbits of the captured particles only, they will fill all the space around the Earth where the existence of its satellites is possible. Since the value of ω_c is less than the angular velocity of a particle in any circular geocentric orbit, then the average specific momentum $l^2\omega_c$ of particles entering the swarm at a distance l is also less than the specific momentum of a satellite in an orbit with radius l in the central plane of the swarm:

$$l^2\omega_c < \sqrt{Gml}. \tag{4.68}$$

The specific momentum $l^2\omega_c$ of material captured at a distance from the Earth equalling the Laplace sphere ($l = r_L = 144\ R$) equals the specific momentum of a direct satellite revolving in a circular geocentric orbit at a distance of $\tilde{\sim}$ 12 R. The momentum of material captured on the equator of the Hill sphere ($l = l_1 \cong 235\ R$) equals the momentum of a satellite at a distance of 78 R. If the material of the swarm were captured /118

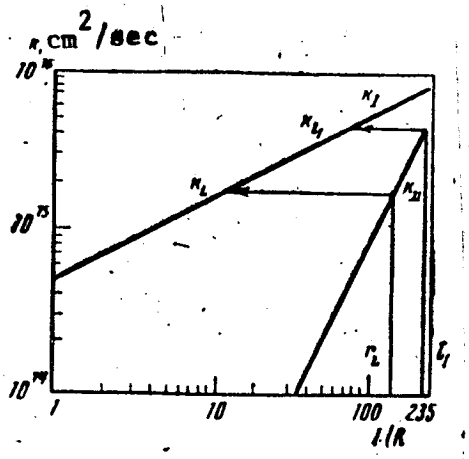

Figure 4.6. Dependence of the specific momentum of satellites of the Earth on the orbital radius l: k_I - specific momentum of satellites in circular orbits, k_{II} - distribution of the specific geocentric angular momentum in the satellite swarm corresponding to its "solid-body" rotation with respect to the Sun; r_L and l_1 - boundaries of the Laplace and Hill spheres respectively; k_L and kl_1 - upper limits of momentum in the Laplace and Hill spheres respectively.

at distances close to 80 R, then its momentum would be less than the specific momentum of a satellite moving in a Keplerian orbit at a distance of 1 R from the center of the Earth, i.e. it would fall to the Earth. For the specific momentum of the Moon at its present orbit, the capture of material must have been accomplished at distances greater than 220 R, i.e. almost at the boundary of the Hill sphere. The distribution of the average specific geocentric momentum in the material captured at various distances is presented in Figure 4.6 (curve k_{II}). The specific momenta $\sqrt{Gml}$ of satellites revolving about the Earth in circular orbits of radius l (curve k_I) are also plotted. The non-correspondance of the first with the second must consistently lead to a "restructuring" of the swarm, indicated in Figure 4.6 by arrows. It is clear that the concentration of captured material toward the planet sharply increases with such restructuring and leads to the fall of a significant mass from the swarm to the Earth. It is possible that this mass exceeded the mass of the Moon by several times and carried with it a significant fraction of the "regular" momentum of the Earth, i.e. the component of its rotational momentum perpendicular to the plane of the ecliptic.

The process of the enrichment of the swarm lasted for almost the entire time of accumulation of the Earth. Fall of material of the swarm to the Earth and its accumulation into one or several protomoons occurred in parallel with this. We introduce the fictitious total (over the entire time of enrichment of the swarm) surface density σ of the captured material only. Let the dependence of σ on the distance from the axis be expressed in the form:

$$\sigma = \sigma_0 (l/R)^{-b}. \qquad (4.69)$$

We estimate the relation of the mass and angular momentum for the two portions of the material of the swarm: for that captured close to its axis of rotation and presumably falling to the Earth and that captured far away, by assumption, combining into the Moon. /119

We denote the boundaries of the inner and outer portions of the swarm by (l_1, l_2) and (l_2, l_3) respectively, where $l_3 > l_2 > l_1$. Then the ratio of the mass M falling to the Earth to the mass μ forming the Moon will be:

$$\frac{M}{\mu} = \frac{2\pi \int_{l_1}^{l_2} \sigma l \, dl}{2\pi \int_{l_2}^{l_3} \sigma l \, dl} = \frac{l_2^{2-b} - l_1^{2-b}}{l_3^{2-b} - l_2^{2-b}}, \tag{4.70}$$

the ratio of the total angular momenta:

$$\frac{K}{K_\mu} = \frac{2\pi\sigma_0\omega_c \int_{l_1}^{l_2} l^{3-b}\, dl}{2\pi\sigma_0\omega_c \int_{l_2}^{l_3} l^{3-b}\, dl} = \frac{l_2^{4-b} - l_1^{4-b}}{l_3^{4-b} - l_2^{4-b}}, \tag{4.71}$$

The specific momentum of the material of the Moon:

$$k_\mu = \frac{K_\mu}{\mu} \frac{\omega_c (l_3^{4-b} - l_2^{4-b})(2-b)}{(l_3^{2-b} - l_2^{2-b})(4-b)}. \tag{4.72}$$

and the specific momentum of the material falling to the Earth:

$$k = \frac{K}{M} = \frac{\omega_c (l_2^{4-b} - l_1^{4-b})(2-b)}{(l_2^{2-b} - l_1^{2-b})(4-b)}. \tag{4.73}$$

the radius of the Earth ($l_1 = R$), can be taken as the inner boundary l_1, and the radius of the Hill sphere for l_3. The boundary l_2 between the two portions of the swarm can be chosen from the condition that the specific momentum $l_2^2\omega_c$ be equal to the specific momentum of a satellite revolving at the Roche limit of tidal stability ($\tilde{\sim}$ 3 R); then $l_2 \approx 100R$. The ratios of mass and momenta, specific momenta and total momentum of the Earth-Moon system corresponding to these boundaries are presented in Table 6 for various exponents of the concentration b. We see that the amount of material falling from the swarm to the Earth increases greatly with an increase in concentration of the captured material toward the center, the ratio of momenta increases more weakly, and the initial specific momentum of the Moon depends even less on the degree of concentration.

For $l_s = 235\,R$ and only one protomoon, the latter revolves at a distance of (15 - 27)R depending on the degree of concentration of the material toward the center. A system of protomoons could have been formed in the region (3 - 78)R. Because of the fall of material from the swarm, the momentum of the Earth K can be obtained both larger and smaller than the present value equalling $5.8 \cdot 10^{40}$ gm·cm^2/sec, but in all the considered cases the total momentum of the Earth-Moon system is obtained less than the present value equalling $3.45 \cdot 10^{41}$ gm·cm^2/sec. To obtain data agreeing with the observations, it is necessary to assume that a significant part of the momentum of the Earth itself was obtained by the direct growth. /120

TABLE 6*

MASS AND ANGULAR MOMENTUM DISTRIBUTION IN THE SWARM BOUNDED BY HILL SPHERE (235 R) FOR VARIOUS EXPONENTS OF CONCENTRATION b

b	M_μ	K/K_μ	$k \cdot 10^{-39}$, cm^2/sec	$k_\mu \cdot 10^{-39}$, cm^2/sec	$K \cdot 10^{-40}$, gm·cm^2/sec ($\mu = \mu_\zeta$)	$K_\mu \cdot 10^{-40}$, gm·cm^2/sec ($\mu = \mu_\zeta$)	$(K + K_\mu) \cdot 10^{-41}$, gm·cm^2/sec	l_ζ/R
0	0,22	0,033	40	2,6	0,065	1,9	2	27
1	0,73	0,083	27	2,4	0,14	1,75	1,9	23
2	5,4	0,22	8,7	2,1	0,34	1,55	1,9	17,4
2,5	25,8	0,38	3	2,0	0,56	1,46	2,1	16
3	172	0,73	0,8	1,9	1,0	1,4	2,4	14

*Translator's note: Commas in numbers represent decimal points.

According to Giuli [25], who considered numerically the scheme of direct growth of the Earth due to particles revolving about the Sun in the plane of the ecliptic, the Earth must have acquired a momentum $K_0 = (1.7 - 1.8) \cdot 10^{41}$ gm·cm^2/sec, i.e. rotation with a period of seven hours. One obtains with the use of the data of Table 6 that for K_0 from $1.7 \cdot 10^{41}$ to $1.8 \cdot 10^{41}$ gm·cm^2/sec, according to Giuli, and for b from 0 to 2.5 corresponding to fall from the swarm to the Earth from several tenths to several tens of lunar masses, the sum of the total momentum of the swarm and initial momentum of the Earth equalling $3.7 \cdot 10^{41}$ gm·cm^2/sec is close to the present momentum of the Earth-Moon system ($3.45 \cdot 10^{41}$ gm·cm^2/sec).

The relative contribution of satellite swarms to the rotation of other planets of the solar system was evidently less (cf. Chapter 6). One can reach the following conclusions from this analysis.

1. The density of the swarm increased strongly in the direction toward the Earth, and a significant part of it must have fallen to the Earth carrying with it a significant part of its rotational momentum.

2. If the boundary of the swarm can be approximated by the Hill sphere (235 R) and if the distribution $\sigma(l)$ can be approximated by formula (4.63), then a single protomoon is formed at a distance of (15 - 27)R, while a system of protomoons is formed in the region (3 - 78)R. The obtained initial distance of a protomoon fits well into the interval of initial distances of the Moon from 10 to 30R, which was obtained by Goldreich from the calculations of tidal evolution of the lunar orbit.

4.6. Dynamic Characteristics of the swarm. /121

Relaxation time of the swarm. Let us evaluate the time τ_s, over which a particle of the swarm (satellites) experiences a collision with smaller particles, also belonging to the swarm, whose total mass equals its mass m_s [26].

We will neglect fracturing of the particles. For small bodies unable to grow, τ_s is a characteristic time for the ordering of the motion in the swarm because of inelastic collisions. For large bodies able to grow, τ_s denotes the time for doubling the mass or a characteristic time of growth, if the mass loss with collisions with particles is small, i.e. the disintegration coefficient (cf. below) $f_s \approx 0$:

$$\Delta m_s \cong m_s = \pi a^2 \rho_s v_s \tau_s. \tag{4.74}$$

Here ρ_s is the density of particles smaller than m_s. Substituting $m_s = {}^4/_3 \pi \delta_s a^3$, $\rho_s v_s = {}^4\sigma_s/P$, where σ_s is the surface density of the material of the swarm, including only bodies with masses smaller than m_s, we obtain:

$$\tau_s = a\delta_s P/3\sigma_s,$$
$$\sigma_s = \frac{\int_{a_{min}}^{a} m_s B_3 a^{-n_3} da}{S} = \frac{\mu_s}{S} \frac{\int_{a_{min}}^{a} m_s B_3 a^{-n_3} da}{\int_{a_{min}}^{a_{max}} m_s B_3 a^{-n_3} da}, \tag{4.75}$$

where μ_s is the mass of the entire swarm equalling the integral in the denominator, whose introduction here in explicit form is convenient,

and S is the area of the swarm. The normalization factor B_3 refers to the total volume of the swarm. Hence, we find for $n_2 \neq 4$:

$$\sigma_s = \frac{\mu_s}{S}\frac{(a^{4-n_2}-a_{min}^{4-n_2})}{(a_{max}^{4-n_2}-a_{min}^{4-n_2})}, \quad \tau_s = \frac{\delta_s PaS\,(a_{max}^{4-n_2}-a_{min}^{4-n_2})}{3\mu_s\,(a^{4-n_2}-a_{min}^{4-n_2})}, \tag{4.76}$$

and for $n_2 = 4$:

$$\sigma_s = \frac{\mu_s}{S}\frac{\ln(a/a_{min})}{\ln(a_{max}/a_{min})}, \quad \tau_s = \frac{\delta_s PaS}{3\mu_s}\frac{\ln(a_{max}/a_{min})}{\ln(a/a_{min})}. \tag{4.77}$$

It is seen from (4.76) that τ_s for $n_2 = 3$ and $a \gg a_{min}$ does not depend on the dimensions of the particles. For $n_2 < 3$, τ_s is smaller for larger bodies than for smaller; for $n_2 > 3$, conversely, the motion of small bodies is ordered more rapidly than the large bodies grow and are ordered. More favorable conditions for the accumulation of large satellites are created in the latter case. The values of τ_s, calculated for $n_2 = 3.5$ and 4 depending on μ_s for $P = 10^6$ sec ($l = 32\ R$) and $S = \pi\ (60\ R)^2$ (the area of the circle bounded by the present lunar orbit), are presented in Table 7. In other orbits l/τ_s differs from that presented by $l^{7/2}$ times, since $P \sim l^{3/2}$; for example, for $l = 60\ R$ the time τ_s is 2.5 times greater. /122

It is seen from the table that the relaxation time is much less than the characteristic time for accumulation of the Earth - 10^8 years [4]. This speaks in favor of the rapid evolution of the swarm.

TABLE 7*

RELAXATION TIME τ_s (IN YEARS) DEPENDING ON μ_s FOR VARIOUS VALUES OF a

$\mu_s/\mu_\oplus$	a, cm					
	10	10^2	10^3	10^4	10^5	10^6
$n_2 = 3,5$, a_{max} 10^6 cm, $a \gg a_{min}$						
10^{-4}	$7\cdot10^3$	$7\cdot10^4$	$7\cdot10^5$	$2,2\cdot10^6$		
10^{-3}	7	70	700	$2,2\cdot10^3$	$7\cdot10^3$	
1,0	0,07	0,7	7	22	70	220
$n_2 = 4$, $a_{max} = 10^6$ cm, $a_{min} = 10^{-4}$ cm						
10^{-6}	48	$3,4\cdot10^3$	$2,7\cdot10^4$	$2,4\cdot10^5$		
10^{-4}	0,48	34	$2,7\cdot10^3$	$2,4\cdot10^4$		
10^{-3}	$4,8\cdot10^{-3}$	0,34	27	240	$2,2\cdot10^3$	
1,0	$4,8\cdot10^{-3}$	$3,4\cdot10^{-3}$	0,27	2,4	22	200

*Translator's note: Commas in numbers represent decimal points.

Disintegration of small satellites of the Earth by particles from interplanetary space. We have seen that with the presence of the ordered circumterrestrial swarm, the large nucleus of a satellite is able to quickly double its mass by scooping up particles from geocentric orbits. Without the swarm, all satellites smaller than some dimension must be fragmented under the effect of collisions of particles from heliocentric orbits. In fact, in approaching the growing Earth, the geocentric velocity of an interplanetary particle is:

$$v_1 = \sqrt{\frac{2Gm}{l} + \frac{Gm}{\theta r}}, \tag{4.78}$$

if it is not slowed in the resisting medium. With respect to the satellite revolving in a circular orbit with a velocity $\sqrt{Gm/l}$, the average velocity of the particle will be:

$$\bar{v}_1 = \sqrt{\frac{2Gm}{l} + \frac{Gm}{\theta r} + \frac{Gm}{l}}. \tag{4.79}$$

In the space bounded by the present lunar orbit, this velocity is included between 14 and 3.5 km/sec, when the mass of the Earth is growing from $M_{pres}/2$ to M_{pres}. This velocity is much greater than the escape velocity from the surface of satellites, which for a body with a density of 3.3 gm/cm^3 and radius 1,000 km is 1.36 km/sec for a body with /123 radius 100 km - 0.136 km/sec, etc. The specific kinetic energy of interplanetary particles, which they must have in colliding with satellites of the Earth, was the order of 10^{11} erg/gm. In such a collision the particle must fragment a much larger mass than its own. To fragment stony bodies requires from 10^8 to 10^9 erg/gm [27], and to overcome the gravitational field of satellites with radii from 10 to 100 km, from 10^6 to 10^8 erg/gm is sufficient. Thus, impacts of interplanetary particles in a majority of cases were accompanied not only by fragmentation of the bodies and the particles themselves, but also the injection of fragments into independent circumterrestrial orbits. Thus, for example, if one takes a fracturing energy of 10^8 erg/gm corresponding to disintegration into large fragments, then a satellite with radius of 100 km must have been fragmented and dispersed with impact of a body with radius of 30 km colliding with it at a velocity of 1.5 km/sec, or a body with a radius of 15 km flying with a velocity of 5 km/sec. In order that a 100 km satellite be fragmented more strongly (an energy of 10^9 erg/gm), the radii of bodies impacting with a velocity of 1.5

and 5 km/sec must be 50 and 25 km respectively, i.e. still significantly less than the radius of the satellite.

Let us evaluate the time for the disintegration of a satellite of radius a_s by various interplanetary bodies impacting with it with the same velocity $\bar{v}_1$, of which all larger than a_d disintegrate the satellite at once, and smaller than a_d gradually chip it away. As was noted above, $a_d < a_s$.

Let $dN(a) = B_1 a^{-n_1} da$, then the time during which the satellite experiences one disintegrating collision on the average, is:

$$\tau_d = \left(\int_{a_d}^{a_{max}} B_1 a^{-n_1} \pi (a_s + a)^2 \bar{v}_1 \, da \right)^{-1}, \tag{4.80}$$

where a_{max} is the maximum radius of the bodies.

The time during which the satellite of mass m_s collides with particles smaller than a_d having a total mass sufficient to fragment it is:

$$\tau_c = m_s \left[f_s \int_{a_{min}}^{a_d} \frac{4\pi}{3} \delta_s a^3 B_1 a^{-n_1} \pi (a_s + a)^2 v_1 \, da \right]^{-1}, \tag{4.81}$$

where the loss coefficient with bombardment f_s indicates by how many times the mass ejected from the satellite by a particle is larger than the mass of the particle itself. The average disintegration time τ_f of a satellite is determined from the expression

$$\frac{1}{\tau_f} = \frac{1}{\tau_d} + \frac{1}{\tau_c}. \tag{4.82}$$

The coefficient f_s depends on the energy of the impacting particle /124
and on the mass of the satellite. In a work devoted to the conditions for growth of asteroids, Hartmann [28] calculated f_s on the basis of the experimental data of Holt and others on impacts with supersonic velocities of 5 - 7 km/sec. In the experiments of Holt, the ratio of the fragmented mass to the mass of the "charge" was 10^2 - 10^3, but high ejection velocities up to 19 km/sec were observed only for a small portion of the material with a mass approximately equalling the mass of the "charge." The lower velocity limit of 0.25 km/sec was recorded for this ejection. This value is the parabolic velocity at the surface of a body with radius 176 km (for a density of 3.6 gm/cm^3). Hartmann took this value as the lower limit for the radius of an in-

destructable asteroid, which is subjected to the impacts of particles with velocities of 5 - 7 km/sec. For smaller bodies, which do not grow but only disintegrate with impacts, he presents values for the coefficient f_s depending on their radii a_s (for a density of the bodies of 3.3 gm/cm^3, the radii would be 5% greater):

a_s, km	176	100	10	1	<1
f_s	>1	4	26	10^2	10^3

A. Marcus [29] presents higher disintegration coefficients on the basis of the same experimental data of Holt and others, but with consideration of the entire velocity distribution function for the ejected material. According to Marcus, for an impact velocity of 5 - 7 km/sec $f_s = 1$ corresponds to a body with a radius of about 400 km; for bodies with radii less than 200 km, the coefficient f_s is greater than 10; for a = 100 km, $f_s \approx 30$; with the transition to smaller bodies, f_s decreases to several hundred.

The data of Holt are applicable to the conditions of the circumterrestrial space, in which the typical velocities of interplanetary particles were about 5 km/sec. Hence, it follows that for the growth of satellites of radius less than some value (400 km according to Marcus), a circumterrestrial swarm of particles with much lower secular velocities would be required. The calculation shows that without a swarm during the period of active growth of the Earth, when the spatial density of interplanetary particles in the supply zone of the Earth was 10^{-11} - 10^{-12} gm/cm^3 and their radius distribution had an exponent n in the interval from 3 to 4, a satellite with radius of 100 km ($f \approx 30$) would have been fragmented after a time the order of 10^5 years. Bodies with radii of 200 km, for which f is somewhat less, would have disintegrated after 10^6 - 10^7 years. Thus, in the vicinity of a sufficiently massive Earth without a circumterrestrial swarm, nobody of asteroid dimensions could have grown into the Moon because of interplanetary bodies and particles, but only fragmented by them.

Growth of large satellites of the Earth because of interplanetary particles. We assume that there is no circumterrestrial swarm, but there are large satellites in the vicinity of the Earth, which have not yet been fragmented with impacts of interplanetary particles, and

are even capable of growing with their collisions. Let us evaluate the maximum growth of their mass by assuming that the disintegration coefficient $f_s \approx 0$. Actually, the real values of f_s are always greater than zero, since the particles cause fragmentation of the surface of the satellites, but their fragments and ejection products remain in circumterrestrial orbits and in the final analysis fall to the satellite and to the Earth. The simultaneous growth of the Earth and the satellite of mass μ and radius a are described by the relations: /125

$$\frac{dm}{dt} = \frac{4\pi}{p} r_e^2 \sigma, \quad r_e^2 = r^2\left(1 + \frac{2Gm}{v^2 r}\right), \tag{4.83}$$

$$\frac{d\mu}{dt} = \frac{4\pi}{p} a_e^2 \sigma, \quad a_e^2 = a^2\left(1 + \frac{2G\mu}{v^2 a}\right). \tag{4.84}$$

Let $v^2 = Gm/\theta r$ as before, then:

$$r_e^2 = r^2(1 + 2\theta), \quad a_e^2 = a^2[1 + 2\theta\,(\mu/m)^{2/3}] \approx a^2. \tag{4.85}$$

By equating the expression for dμ/dm with that obtained by dividing (4.84) by (4.83), we obtain:

$$\frac{d\mu}{dm} = \frac{4\pi\delta_s a^2 da}{4\pi\delta_\oplus r^2 dr} = \frac{a^2}{r^2(1 + 2\theta)}, \tag{4.86}$$

i.e.:

$$\frac{da}{dr} = \frac{\delta_\oplus}{\delta_s} \frac{1}{1 + 2\theta}. \tag{4.87}$$

for $\delta_\oplus/\delta_s = 2/3$ and $\theta = 3$, this gives da/dr = 5/21; for $\theta = 5$, da/dr = 5/33.

Consideration of the spatial consolidation of the orbits of particles moving close to the Earth can somewhat increase the effective cross section πa_{eff}^2 of satellites located close to the Earth [30]. However, in this case the mass lost by the satellite increases at the same time because of the great intensity of impacts, thus our conclusion remains valid.

Only those satellites could be preserved, which began growing when the mass of the nucleus of the Earth had reached approximately half the present terrestrial mass (with a smaller mass of the nucleus of the Earth, there is no guarantee that the system would not have been fragmented by impacts of other large masses in the supply zone of the Earth). Moreover, the initial mass of the nucleus of the planet is small, then with its further significant increase all satellites initially located at a distance $l_0 < l_R M/m_0$ are pulled inside the Roche limit l_R and disintegrate. With the growth of the Earth from $M_{pres}/2$ to

M_{pres}, its radius increases by 1300 km. According to (4.87), the growth of the radius of a large satellite is a maximum of 200 - 300 km. To provide for the growth of the Moon by this way, it would be necessary that for a mass of the Earth twice as small as now that there already existed a protomoon with a radius of 1,400 - 1,500 km, i.e. a satellite /126 with a mass equalling 0.5 - 0.7 the present mass of the Moon, or 3 or 4 protomoons, each with a radius of 100 km and the same total mass. Consequently, although the growth of large satellites because of interplanetary particles is possible, this does not solve the problem of the formation of the satellites, which themselves must grow because of the swarm.

Conditions for the growth of a satellite in the circumterrestrial swarm. Let us evaluate what the circumterrestrial swarm must be, in which the conditions for growth of a satellite are satisfied. The averate velocities $\bar{v}_s$ of the particles of the swarm with respect to the satellite moving in a circular orbit were determined by the inclinations of the orbits of the particles to the orbit of the satellite:

$$\bar{v}_s \cong \bar{e} v_c \cong \bar{i} v_c, \tag{4.88}$$

where v_c is the circular geocentric velocity equaling $\sqrt{Gm/l}$, e is the eccentricity of the orbit of the particle, i is the inclination of its orbit to the orbit of the satellite in radians. For a sufficient ordering of the motion of the particles of the swarm $e \ll 1$, $i \ll 1$ and the velocities v_s are much less than v_1 given by (4.79). The ordering of the motion was achieved over a comparatively short time, as was shown above. The velocities v_s are uniquely related to the uniform thickness of the swarm $v_s = 4H_s/P$, where P is the geocentric period of revolution of the particles of the swarm. The degree of ordering or the consolidation of the circumterrestrial swarm was limited by the presence of large bodies in it, whose perturbations also determined $\bar{e}$ and $\bar{I}$:

$$\bar{e} = \bar{i} \approx \frac{\bar{v}_s}{v_c} = \frac{\sqrt{G(m_s)_{max}/\theta a_{max}}}{\sqrt{Gm/l}} = \frac{a_{max}}{r}\sqrt{\frac{l\delta_s}{r\theta\delta_\oplus}}. \tag{4.89}$$

It is these largest bodies at this time which were capable of further growth because of the particles of the swarm, i.e. they could become nuclei of satellites, and the smaller bodies served only as the supply medium for them.

The possibility of the growth of the large bodies in the vicinity of the growing Earth depended on the ratio of the densities of the circumterrestrial swarm and the interplanetary medium. Two components of the swarm should be distinguished in analyzing this process. One consists of particles in ordered orbits; the other of particles newly captured in the swarm, whose orbits are at first greatly elongated and inclined to the central plane of the swarm, where large satellites must be accumulated. We call these particles the unordered component of the swarm. The growth in mass of the satellite is:

$$\frac{dm_s}{dt} = \pi a_s^2 [\rho_s v_s (1 - f_s) + \rho_i v_i (1 - f_i) + \rho_1 v_1 (1 - f_1)], \qquad (4.90)$$

where f_s is the loss coefficient with collision with the ordered component of the swarm, ρ_s is its density, f_1 and f_i, ρ_i and ρ_1, v_i and v_1 are respectively the loss coefficient, density and velocity of particles of the unordered component and the interplanetary particles. The total density of the swarm ρ_2 is the sum of ρ_s and ρ_i. For an approximate estimate, we assume that f_i and f_1 are the same and equal to f, and since $\bar{v}_s \ll \bar{v}_1$, we assume f_s equals zero and let $\bar{v}_i \approx \bar{v}_1$, $\rho_i \approx \rho_1$. Then the condition for the growth of a satellite $dm_s/dt > 0$ is written in the form: /127

$$\rho_s \bar{v}_s > 2 \rho_i \bar{v}_i (f - 1) \qquad (4.91)$$

or with substitution of:

$$\frac{\bar{v}_i}{\bar{v}_s} = \frac{r}{a_{max}} \sqrt{\frac{\delta_\odot}{\delta_s}\left(1 + \frac{3\theta r}{l}\right)}$$

in the form:

$$\frac{\rho_s}{\rho_i} = 2 \frac{r}{a_{max}} (f - 1) \sqrt{\frac{\delta_\odot}{\delta_s}\left(1 + \frac{3\theta r}{l}\right)}, \qquad (4.92)$$

where the value of the root changes from approximately 2.5 for l to 1.4 for large l. It makes sense to consider this ratio close to the plane of symmetry of the swarm, where the density ρ_s is maximum and where the nuclei of satellites revolve. We will consider this condition as applied to nuclei of satellites of various dimensions a_s, keeping in mind bodies which are the largest in this zone of the swarm and thus determine the value of v_s.

Satellites larger than some limit, for which $f \leq 1$, grow for any $\rho_s > 0$. According to Marcus, this limit is 400 km; according to Hartmann, the limiting radius is 2 times smaller. But for smaller satellites,

the condition leads to rather rigid requirements on the ratio of ρ_s. Thus, for example, for satellites with radii of 100 km with velocities v_s determining their effect, we have $2\ r/a_s \gtrsim 100$ and, moreover, f is several ten's. Thus, a density of the ordered component of the swarm 3 orders of magnitude greater than the density of the interplanetary medium is required for the growth of a 100-km satellite. The growth of a smaller nucleus requires not only a higher density in the plane of symmetry but also a larger mass of the swarm. In fact, the condition is a lower limit for the surface density of the swarm, if it is rewritten in the form:

$$\sigma_1 > \frac{p}{2}\rho_1 \bar{v}_1 (f-1). \qquad (4.93)$$

Hence, one can conclude that the current mass of the swarm proportional to σ_s must be larger, the smaller the initial dimension of the nucleus, since the coefficient f increases with its decrease. Thus, the growth of a satellite from a sufficiently large body captured into the swarm with collisions of similar bodies is more realistic, since the possibility of growth of a satellite beginning with a nucleus of arbitrarily small radius is doubtful.

The calculation of the number of collisions of bodies of various dimensions in the vicinity of the Earth over the time of its growth, which could lead to the capture of a satellite, indicates that the probability of one collision of two bodies with radii of 1000 km is less than 10^{-3} (cf. Chapter 3.3). The same calculation indicates that /128
over this time could occur single collisions of bodies with radii of 200 - 300 km and several tens of collisions of bodies with radii of 100 km (if the main mass of the material of the supply zone of the Earth was concentrated in such bodies). This indicates that the maximum initial dimension of satellites of the Earth, captured into the swarm, was probably no greater than 100 km, and only sufficiently large fragments could have survived with the collisions of bodies. The growth of such satellites of the Earth would have required a circumterrestrial swarm with an initial density in its central plane 100 times greater than the density of the interplanetary medium. This requirement is satisfied with the exponential growth of the mass of the swarm according to Formula (4.61), when it is not exhausted.

One can conclude from the above discussion that the active stage of accumulation of the Earth must have been accompanied by the formation of a curcumterrestrial satellite swarm due to inelastic collisions of particles and bodies in the gravitational sphere of the Earth. Some portion of the material of this swarm must have fallen to the Earth, but the rest must have remained in geocentric orbits with direct motion.

The pattern of growth of satellites in the swarm can be divided qualitatively into two stages: an initial monotonic enrichment of the swarm because of "free-bound" collisions of particles, enrichment of its small fraction, ordering of the motion of the particles, contraction of the swarm inward, achievement of a high density ρ_s in the plane of symmetry of the swarm at comparatively small distances from the planet; then the beginning of growth of the largest bodies with scooping up of the ordered component of the swarm. In the second stage the density of the small fraction had evidently not grown, but been maintained at some level. The condition $\bar{\rho}_2 \geq 10\ \rho_1$ (4.67) is sufficient for the passage of about 10 lunar masses through the swarm, if it were completed on the average over the entire time of growth of the Earth. It could have been "over-satisfied" in the first stage, since there was no scooping up, and "under-satisfied" in the second stage, since the growth of satellites was accompanied by the acceleration of scooping up. Each large body was a potential nucleus of a satellite; however, in connection with the great probability of disintegration as a result of impacts of particles and bodies passing with heliocentric orbits, only the largest bodies with radii the order of 1000 km could have survived in the circumterrestrial swarm.

This consideration indicates that the formation of the circumterrestrial swarm and the growth of large satellites in it are "recorded" in a natural manner in the scheme of the growth of the Earth.

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Chapter 5

ACCUMULATION OF THE MOON IN THE CIRCUMTERRESTRIAL SWARM

The brevity of the processes in the circumterrestrial satellite swarm ($\sim 10^3$ years) and the duration of the time of accumulation of the Earth ($\sim 10^8$ years) evidently exclude a monotonic growth of the Moon from a single nucleus. It is likely that its growth was accompanied by collisions from the very beginning, and not just at the end when the visible relief was formed. A system composed of the Earth and several protomoons should be considered as one of the possible prehistories of the Earth-Moon system. By protomoons should be understood only the largest satellites, capable of existing more than 10^8 years with their bombardment from heliocentric orbits, i.e. bodies with dimensions the order of thousands of kilometers. Only the participation of large bodies in the accumulation of the Moon gives the higher initial temperature of its interior and can explain the inhomogeneities of its structure [1]. The peculiarities of the chemical composition of the Moon can be related to the mechanical processes in the formation of the swarm. The subsequent history of the lunar atmosphere is also related to the sorting of volatile substances with the formation of the Moon. /130

5.1. Maximum and Most Probable Number of Large Satellites of the Growing Earth

Let us evaluate the possible number of large protomoons forming near the growing Earth. The maximum number of large satellites of the Earth can be evaluated by knowing the width of the bodies in circular zones of the swarm, where these satellites dominated the other bodies, under the assumption that the orbits of the satellites themselves were circular. By analogy to that assumed by V.S. Safronov for the supply zones of the growing planets [2], we can assume that the width of the zone is proportional to the average eccentricity of the orbits of the supply particles:

$$\Delta l_s = \lambda 2el, \qquad (5.1)$$

where l is the radius of the circular orbit of the satellite. With an eccentricity of the satellite orbit, the supply zone must be wider.

The eccentricities, as well as the inclinations of the orbits of the supply particles, are related to perturbations of the satellite of mass μ and radius a: /131

$$e \approx i \approx v/v_c,$$

where:

$$v = \sqrt{G\mu/(\theta a)}, \quad v_c = \sqrt{Gm/l},$$

from which:

$$\Delta l = 2\sqrt{\mu/(\theta m a)}\, l^{3/4}. \tag{5.2}$$

Figure 5.1 shows the dependence of Δl on l for 4 cases of satellite systems with masses $\mu = {}^1/_{10}\mu_\zeta$, ${}^1/_8\,\mu_\zeta$, ${}^1/_3\,\mu_\zeta$ and ${}^1/_2\,\mu_\zeta$ for $\theta = 3$ in the space bounded by the present lunar orbit. We see that the width of the supply zones increases rapidly with distance from the Earth.

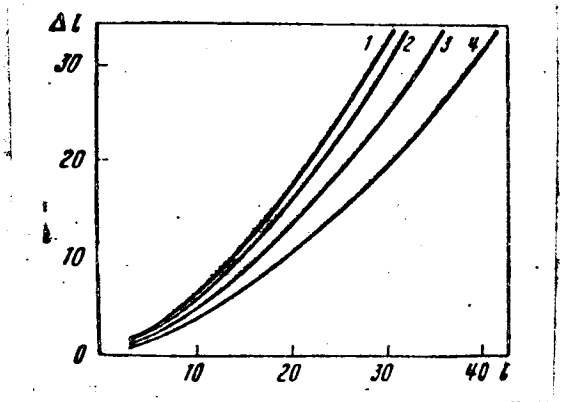

Figure 5.1. Dependence of the width of the supply zones of satellites on their distance from the Earth:
1 — $\mu = {}^1/_{10}\mu_\zeta$;
2 — $\mu = {}^1/_8\mu_\zeta$;
3 — $\mu = {}^1/_3\mu_\zeta$;
4 — $\mu = {}^1/_2\mu_\zeta$

The arrangement of the satellites in their own zones depends on the concentration of material toward the center of the swarm. It can be evaluated for satellites, for which there is no tidal interaction with the central body during the growth process. We introduce, as previously, (cf. Chapter 4) some effective surface density of the swarm σ, which is the time integral of the actual surface density of the swarm at a given distance. Let σ be distributed in the swarm according to the law:

$$\sigma \sim l^{-q}. \tag{5.3}$$

Then the mass of material included in an annular zone with boundaries l_1 and l_2, will be:

$$\mu_1 = \int_{l_1}^{l_2} 2\pi\sigma l\, dl,$$

and the total geocentric momentum of the material of the zone:

$$K_1 = \sqrt{Gm}\int_{l_1}^{l_2}\sqrt{l}\, d\mu = \frac{\text{const}\,\sqrt{Gm}}{{}^5/_2 - q}\left(l_2^{5/2-q} - l_1^{5/2-q}\right). \tag{5.4}$$

The latter determines the position l_s of the satellite forming in the zone: /132

$$K_{\mu_s} = \mu_s \sqrt{Gml_s} = K_s,$$

where μ_s is the mass of the satellite. Hence:

$$\sqrt{l_s} = \frac{(2-q)\,(l_2^{5/2-q} - l_1^{5/2-q})}{(5/2-q)\,(l_2^{2-q} - l_1^{2-q})} \quad \text{for} \quad q \neq 2 \text{ и } 2{,}5, \tag{5.5}$$

$$\sqrt{l_s} = \frac{\sqrt{l_2} - \sqrt{l_1}}{\ln (l_2/l_1)} \quad \text{for} \quad q = 2. \tag{5.6}$$

The outer boundary of the zone has a greater effect on the resulting distance l_s for q<2, and the inner boundary for q>2. For q = 2, we obtain l_s close to the geometric mean of the zone. It was noted in Chapter 4.5 that satisfactory agreement with the present momentum of the Earth-Moon system is obtained if one assumes that the surface density of just the captured material satisfies the relation:

$$\sigma_0 \sim l^{-b}, \tag{5.7}$$

where $1 \leqslant b \leqslant 2.5$. The transition of captured particles into circular orbits with direct motion indicates a decrease in the average distance of the particle from l_0, at which it began revolving around the Earth, to the distance l according to the formula:

$$l_0^2 \omega_c = \sqrt{Gml}, \tag{5.8}$$

from which $l = \text{const}\, l_0^4$. With such "restructuring" of the swarm, its density changes according to the law:

$$\sigma = \frac{\sigma_0}{4}\left(\frac{l_0}{l}\right)^2. \tag{5.9}$$

substituting l_0 from (5.8), we have:

$$\sigma = \sigma_0 l^{-1/2} \sim l^{-b-1/2}, \quad q = b + \frac{3}{2}. \tag{5.10}$$

We note that the condensation of the swarm, which is caused by the growth in mass of the Earth does not change the concentration exponent, since all distances of particles are contracted by the same number of times. It follows from (5.7) and (5.10):

$$2{,}5 \leqslant q \leqslant 4. \tag{5.11}$$

For such values of q, the position of the satellite in its zone is close to the inner boundary l_1. This implies that the protomoon closest to the Earth must have been located in the immediate vicinity of the Roche limit (2.89 R for a liquid satellite and about 2.8 R for a solid satellite of radius $\geqslant 10^3$ km). By rounding off, we can set the distance of protomoon No. 1 equal to 3 R. Then, by making use of Relation /133

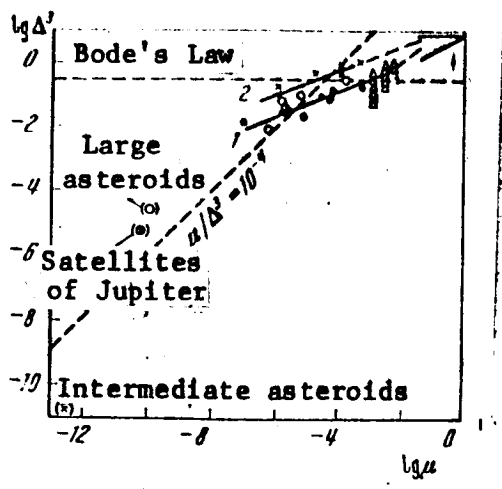

Figure 5.2. Diagram of the masses and relative distances for the observed multiple systems according to G.P. Kuiper. The various systems of protomoons are denoted by triangles on the figure.

(5.2) for the width of the supply zones of the protomoons, one can find the sequence of distances of successive protomoons having adjacent supply zones. One can arrange within the sphere of influence of the Earth no more than 7 protomoons with masses 1/2 $\mu_\oplus$, or 9 with 1/3 $\mu_\oplus$, or 10 with 1/5 $\mu_\oplus$, or 11 with 1/10 $\mu_\oplus$. One can arrange within the present lunar orbit 6-9 protomoons with the selected masses respectively. The number of such protomoons should be considered maximum, since we have neglected the eccentricities of the satellite orbits and their tidal withdrawal in the process of growth.

Let us compare these formal systems with actual multiple systems of celestial bodies. One can use for this the diagram compiled by Kuiper in 1949 for all observed multiple systems [3]. Figure 5.2, which is borrowed from this work, presents on a logarithmic scale the dependence of Δ^3 on the average mass of the satellites μ:

$$\Delta = \frac{l_2 - l_1}{1/2\,(l_2 + l_1)}, \tag{5.12}$$

where l_1 and l_2 is the distance of the satellites from the central body.

If one constructs a single (very rough) dependence for all the bodies of the solar system, beginning with the small asteroids and ending with the large planets, then it forms a straight line on the Kuiper graph, corresponding to:

$$\mu/\Delta^3 = 10^{-4}. \tag{5.13}$$

But if one excludes the asteroids and the irregular satellites of Jupiter, then it appears that a larger part of the planets and satellites satisfies the relation:

$$\lg \mu = 2{,}5 \lg \Delta^3 - A, \tag{5.14}$$

where A = 1.8 for the planets (satellites) of comparable mass (curve 1) and 3.6 for the planets (satellites) greatly differing in mass (curve 2). The band for the corresponding interval of values of A forms a /134

sequence on the diagram, whose extension into the upper right corner also includes miltiple stars (cf. shaded region of Figure 5.2).

Bode's law on the Kuiper diagram corresponds to a horizontal straight line, which reflects significantly more poorly the observed distances. The models of systems of protomoons with adjacent supply zones do not fall onto the presented sequence, the closest protomoons being farthest of all from it. Either a very significant decrease in masses of the satellites (by 3 orders of magnitude), or a comparatively small (by 2 or 3 times) increase in the distances between them would be required for coincidence with actual systems. Thus, if one takes every other protomoon, then their systems are moved close to the main sequence on the Kuiper diagram in just that portion, which refers to a system of satellites of comparable mass. Thus, we could obtain a system of 4 protomoons with masses $1/2\mu_{\zeta}$ and distances of 3, 6, 14.5, 50; or 4 protomoons with masses $1/3\mu_{\zeta}$ at distances of 3, 5.5, 11.5, 31; or 5 protomoons with masses $1/5\mu_{\zeta}$ and distances 3, 5, 9, 22, 60; or 6 protomoons with masses $1/10\mu_{\zeta}$ in orbits with radii of 3, 4.5, 7, 12, 25 and somewhat greater than 60 R. We see that there are many possibilities for constructing a system of satellites of the Earth, which are arranged within the present lunar orbit, with a total mass close to the mass of the Moon and in a dynamic relation similar to actual satellite systems.

5.2. Dynamic History of a Double-Satellite System of the Earth as an Example of the Prehistory of the Earth-Moon System

Further limitation of the possible number of protomoons results from the fact that the later merging of the large masses must have occurred before the generation of the relief observed at present on the Moon. The largest-scale formation on the Moon, undoubtedly of impact origin, is Mare Imbrium. Its structure indicates that the body impacting on the Moon came from a circumterrestrial orbit and had a radius of about 150 km. The age of the excavation of Mare Imbrium is estimated as approximately $3.9 \cdot 10^9$ years (cf. Chapter 1.2). The Moon as a single body has existed no less than $4.0 \cdot 10^9$ years. In the epoch $(3.9 - 4.0) \cdot 10^9$ years ago the Moon was significantly closer to the Earth than now. For the different versions of the change

in time of the tidal lag angle (cf. Figure 2.3), the distance of the Moon from the Earth was at the same time no more than 30 R. Hence, it follows that at distances of about 30 R and farther from the Earth, there could have existed only small asteroid-type satellites, and the orbit of the farthest large protomoon must have been in the range (20-25) R. This limits the possible number of large protomoons with masses $(1/2 - 1/3)\mu_{\zeta}$ to only 3 at most or even to 2, if one takes into account the transient existence of a protomoon in an orbit of radius 3 R (the most rapid tidal withdrawal). There also could not have been more than 3 or 4 smaller protomoons with masses $(1/5 - 1/10)\mu_{\zeta}$. /135

In order to more clearly understand the tidal evolution of a system of several satellites around the Earth, there is interest first of all in considering the dynamic history of a system consisting of the Earth and two large satellites with masses the order of $\mu_{\zeta}/2$. A specific example of this type was investigated by A.S. Syzdykov, Ye.V. Nikolayeva and the author [4,5].

It is well known that for each of several satellites causing tides on the planet, only their interaction with the tidal bulges caused by themselves has significance, and the cross effect of the satellites on each other is small in the general case. The exceptions are the cases of satellites having periods with small ratios, for example, of the type 1 : 2, 2 : 3, 3 : 4 (but not 1 : 1!). Only in these cases can the tidal evolution of the orbits proceed under special conditions with conservation of the similarity of the system. We will discuss this effect in more detail in the following chapter. Here we will consider the general case, when tidal friction acting independently on each of two satellites causes the nearer satellite to overtake the further, until their periods approach the approximate ratio 1:1. By an approximate ratio of this type, we mean the state when the satellites are moving in orbits with approximately the same period (their frequencies ω_1 and ω_2 differ by an amount $\Delta\omega$, where $\Delta\omega \ll \omega_1$, $\Delta\omega \ll \omega_2$). The main factor in the evolution of the system then becomes their mutual perturbations with close transits. We will change from the long tidal time scale to the short scale measured by the periods of revolution of the satellites around the Earth. The effect of tidal friction can be neglected on the short scale.

We consider for simplicity the motion of two satellites in one plane xy, whose origin is at the center of the Earth. We assume that all three bodies of the problem - the Earth and the two satellites - are material points, where the mass of the Earth is taken as unity, the masses of the satellites as m and m' < 1. Then the system of equations of motion in vector form can be written in the form:

$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{G(1+m)\mathbf{r}}{r^3} + Gm'\left(\frac{\mathbf{r}'-\mathbf{r}}{\Delta^3} - \frac{\mathbf{r}'}{r'^3}\right),$$
$$\frac{d^2\mathbf{r}'}{dt^2} = -\frac{G(1+m')\mathbf{r}'}{r'^3} + Gm\left(\frac{\mathbf{r}-\mathbf{r}'}{\Delta^3} - \frac{\mathbf{r}}{r^3}\right), \qquad (5.15)$$

where r and r' are the radius vector of the masses m and m'; $r^2 = x^2 + y^2$; $r'^2 = x'^2 + y'^2$; $\Delta^2 = (x - x')^2 + (y - y')^2$; Δ is the relative distance between the bodies m and m'. As is well known, the system of equations of motion for three finite masses can be solved only in numerical form. /136

Several cases of initially unperturbed circular motion of satellites in close orbits, when the difference in radii of the orbits does not exceed the radius of the sphere of influence of each of the satellites:

$$r - r' \leqslant l = rm'^{\frac{2}{5}} \approx 0{,}13\ r. \qquad (5.16)$$

were investigated numerically with the help of the BESM-4 computer.

The initial position of the satellites in the orbits was usually specified so that the masses m and m' formed a right triangle with the central mass located at the vertex of the right angle, in order that the satellite in the near orbit could overtake the more slowly moving far satellite (Figure 5.3). The distance between the satellites at first was then much greater than the radius of the sphere of influence of each:

$$\Delta_0 \gg l. \qquad (5.17)$$

The distance between the satellites changed during the motion, where close transits became possible.

The motion of a pair of satellites with the same masses m = m' = $1/2m_{\oplus}$ was investigated during several orbits, which made it possible to approximately evaluate the perturbations of the orbits and the lifetime of such double-satellite systems.

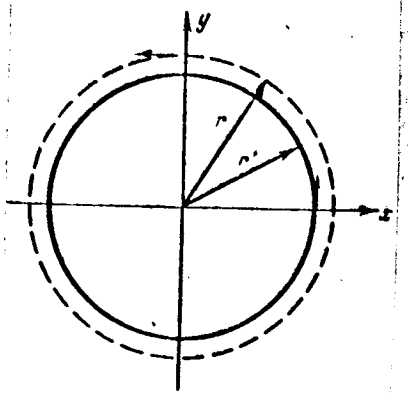

Figure 5.3. Diagram of the initial conditions of motion.

The calculations were carried out with a constant increment, but close approaches were recalculated with a variable increment. Of course, the error δ accumulated with the large number of increments, since $\delta \sim n^{3/2}$ [6], where n is the number of increments, but it had an insignificant effect on the final result. The errors in the calculation can be related in part to the choice of initial conditions, since they were taken arbitrarily. The calculations were carried out for various initial data satisfying the condition (5.17). Figure 5.4 and 5.5 present the results of the calculations for four initial values of the radii of the orbits of the satellites:

$$1)\ r = 4R,\ r' = 4{,}5R;\ \ 2)\ r = 8R,\ r' = 8{,}5R;$$

$$3)\ r = 10R,\ r' = 9{,}5R;\ \ 4)\ r = 15R,\ r' = 14{,}5R.$$

In all cases, the initially nonintersecting circular orbits of the satellites changed into elliptical intersecting orbits after one or two orbits because of the mutual perturbations. In cases 1, 3 and 4 close approaches at distances approximately equaling the sum of the radii of the bodies were obtained over the calculation time, which must correspond to a physical collision of the satellites, if the motion of extended masses differs little from the considered motion of point masses. Neither ejection from the system nor fall to the Earth was found for cases 1 and 2. One can evaluate the probable time, during which the satellites experience a physical collision, from the frequency of intersections of the orbits. We use the empirical formula; /137

$$\tau_c = \alpha \bar{p} \frac{2\pi \bar{r}}{d}. \qquad (5.18)$$

Here α is the number of orbits after which one intersection of the orbits occurs; $\bar{p}$ is the average period of revolution of the satellites; $\bar{r}$ is the average radius of the orbits; d is the diameter of the satellite. One can determine α graphically for the first two versions; for the versions with more separated orbits, one can use the formula:

$$\tau_c = (\tau_c)_1 (r/r_1)^{3/2}, \qquad (5.19)$$

where $(\tau_c)_1$ is the collision time at a unit distance. Estimates of

the times between collisions in the plane case are presented below:

$\bar{r}/R$	4	6	8	10	15
α	0,3	0,4	0,4	—	—
$\bar{P}$, min	50	90	150	200	370
τ_c	1 day	35 day	0,2 year	0,3 year	0,9 year

The evolution time in the three-dimensional case is 2 or 3 orders of magnitude greater; however, it will be $\sim$ 100 years, which is small as compared to the tidal time scale of 10^6 - 10^8 years [7].

Thus, it is seen with the help of the analyzed specific example of the evolution of a double-satellite system that for an approximate simple ratio of the periods of 1:1, the lifetime of such a system is small compared to the duration of its tidal evolution. Consequently, we have the right to limit the consideration to the mutual gravitational perturbations of the satellites only. The main result of the evolution of the orbits included within 20 R, as we have already seen, is the collision of the satellites with velocities approximately equaling the parabolic velocity at their surface, i.e. $\sqrt{2Gm_i/a_i}$. Ejection of one of the satellites from the sphere of influence of the Earth or the fall to Earth of one of the satellites is possible as a particular case of a strong perturbation only with close approaches of the satellites in orbits with a semimajor axis of (16 - 20) R and will not occur at smaller distances.

We estimate the release of energy with the merging of satellites /139 with comparable masses μ_1 and μ_2, when $\mu_1 + \mu_2 = \mu$. The merging of such large bodies is an impact with a significant velocity determined by their gravitational interaction. The sum of the impact energy E_i and the rearrangement energy E_r of the two masses μ_1 and μ_2, which is converted mainly to heat, can be evaluated from the following arguments. The total energy of the system of the two homogeneous spherical masses μ_1 and μ_2 located at an infinitely great distance at first at rest with respect to each other equals the sum of their potential energies as spheres:

$$E_{12} = U_{10} + U_{20} = -\frac{3}{5}\frac{G\mu_1^2}{r_1} - \frac{3}{5}\frac{G\mu_2^2}{r_2}. \qquad (5.20)$$

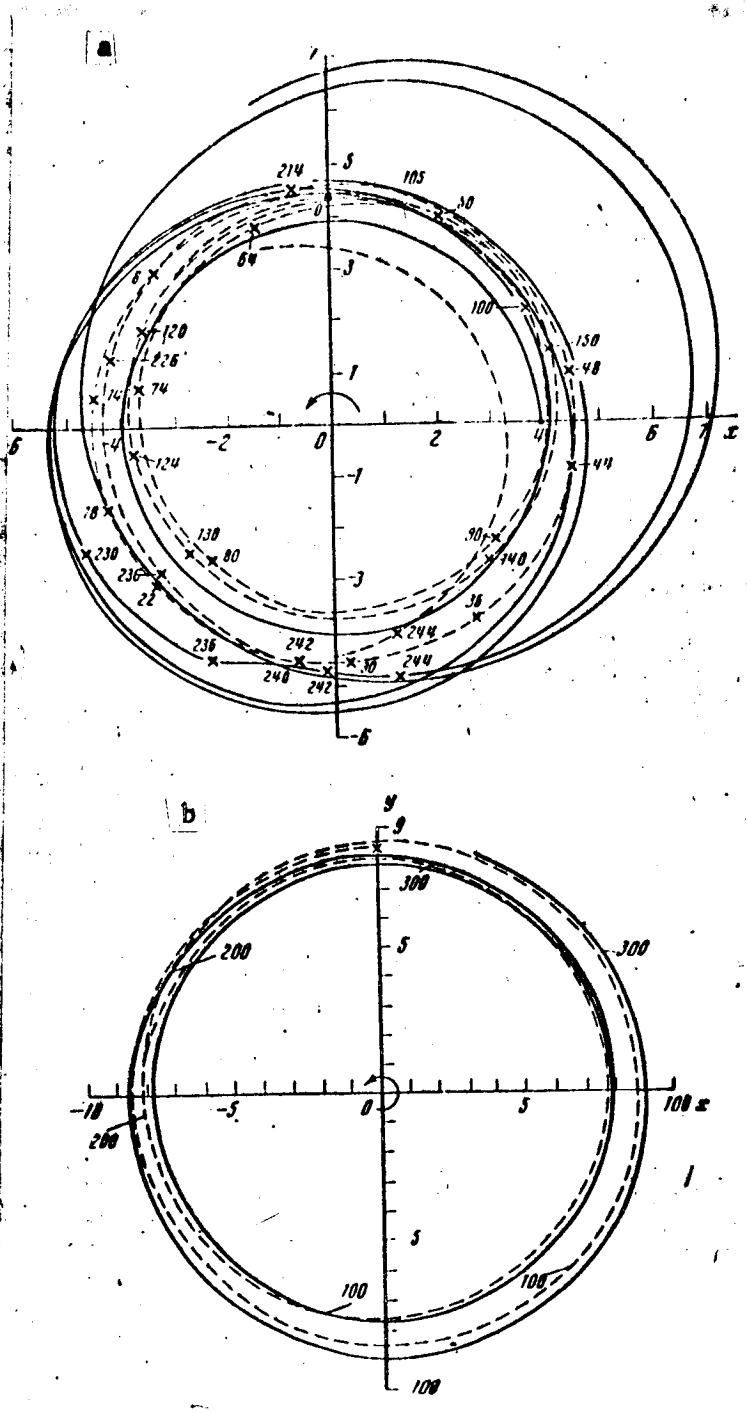

Figure 5.4. Plane motion of a double-satellite system for $m = m' = \mu_c/2$ $a - r_0' = 4R, r_0 = 4{,}5R$; $6 - r_0' = 8R, r_0 = 8{,}5R$. Digits on the orbits - increment number.

With the collision, disintegration and merging into one homogeneous sphere with the mass $\mu_1 + \mu_2$, the kinetic energy of motion and the mass rearrangement energy are expended in the disintegration, heating and radiation from the surface. The total energy of the newly formed sphere is:

$$E = -\frac{3}{5}\frac{G(\mu_1+\mu_2)^2}{r}. \qquad (5.21)$$

The difference of E_{12} and E is the unknown sum of the energies:

$$E_i + E_r = \frac{3}{5}\left[\frac{G(\mu_1+\mu_2)^2}{r} - \frac{G\mu_1^2}{r_1} - \frac{G\mu_2^2}{r_2}\right]. \quad (5.22)$$

The kinetic energy of motion of the bodies "at infinity" should also be added here if it is not equal to zero. The collision of bodies with masses about 1/2 - 1/3 the mass of the Moon occurs with a velocity of about 2 km/sec, whereas impact with a velocity the order of 0.2 km/sec is sufficient for the disintegration of stony bodies. Energy losses to radiation with the collision of two moon-like masses are small, since the time for impact and unification is less than one hour. We can assume that the sum $E_i + E_r$ is expended mainly in heating the entire mass.

For $\mu_1 = \mu_2 = \mu_\zeta/2$ $E_i + E_r = 0{,}37|E|$. The corresponding increase in temperature is about 600°C. In the case, when the merging of a mass $\mu_1 = \mu_\zeta/3$ with an equal mass μ_2 occurs at first, and then a mass μ_3 also equaling $\mu_\zeta/3$ combines with them, and the heat is not able to escape, the increase in temperature is about 900°C. If one takes the equilibrium value of about 300°K for the initial temperature of the bodies, then as a result of the merging of large protomoons, one could, in principle, obtain a temperature for the interior of the Moon of 900°K and higher.

The problem of the "joining" of two large bodies is deserving of a more detailed investigation, since it could help one to understand the inhomogeneities of the structure and the high initial temperature of the newly formed mass. /141

5.3. Differences in the Chemical Composition of the Moon and Earth

The difference in the average densities of the Earth (5.52 gm/cm^3) and the Moon (3.34 gm/cm^3) indicates that their global chemical composition is not the same, since only a difference in the pressures inside these bodies could not alone give such a large difference in densities. If the core of the Earth consists mainly of iron, then it is about 35% of the entire Earth, whereas no more than 14-15% of the Moon [8]. There couldn't be any significant iron core in the Moon (cf. Chapter 1.4). If the core of the Earth consisted of the metallized

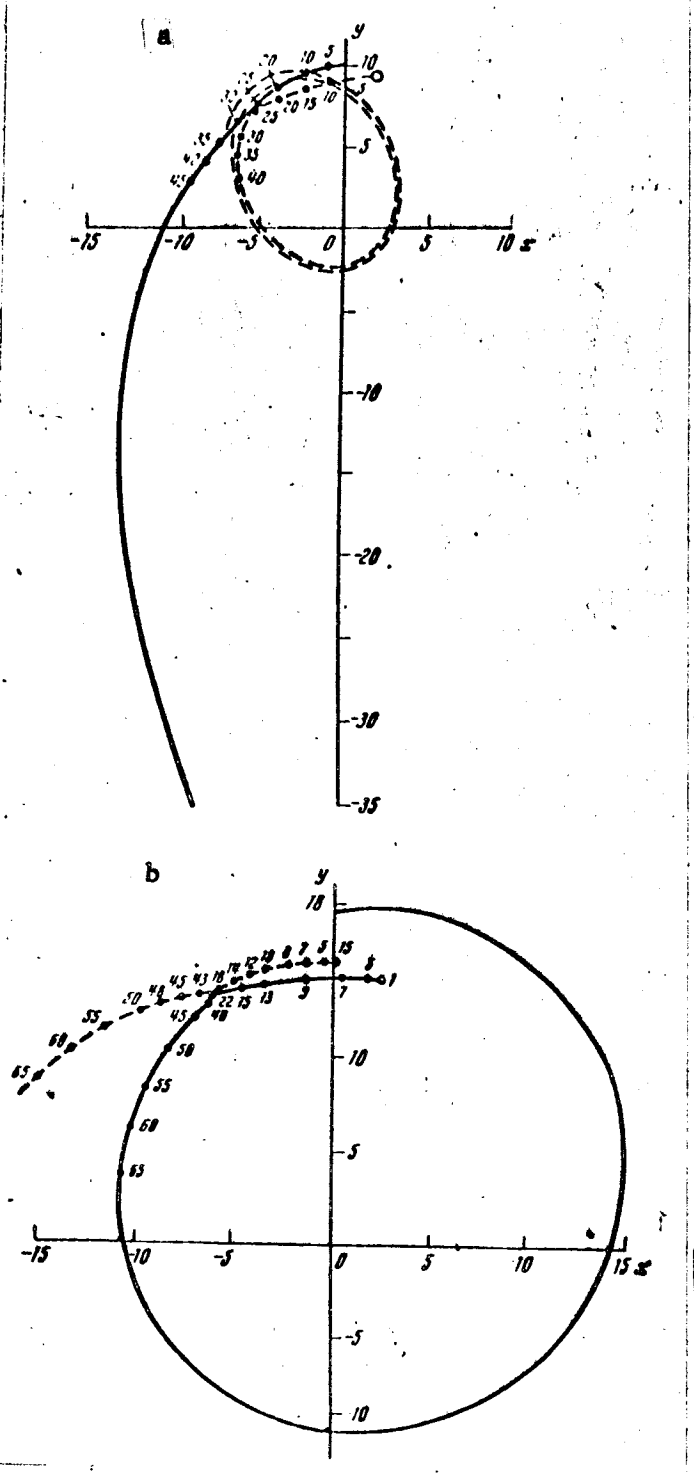

Figure 5.5. Example of the approach of satellites with identical masses ($m = m' = \mu_\oplus/2$) a — $r_0 = 9{,}5R$, $r_\oplus = 10R$; б — $r_0 = 14{,}5R$, $r_\oplus = 15R$. The remaining notation the same as in Figure 5.4.

material of the mantle, then the relative iron content in both bodies could be almost the same. However, differences in the content of the volatile and high-melting elements would still remain in the Moon and in the Earth (cf. Chapter 1.2). With a silicate core of the Earth, the iron content in the Earth-Moon system would be the minimum of the terrestrial planets, and thereby the monotonic decrease in the amount of iron from Mercury to Mars, which is observed with the iron core of the Earth, would be violated [9,10]. There is as yet no reliable experimental data attesting in favor of a silicate core of the Earth. Thus, the problem of the difference in the chemical composition of the Moon and the Earth with an iron core and with different contents of the volatile and high-melting elements still remains open. This problem is a part of the more general problem of explaining the differences in the chemical composition of the satellites and planets. It is likely that there will not be found a single case of complete coincidence of composition of a satellite and its planet. We will turn again to this problem in the sixth chapter.

In this section, we will consider the difference in composition of the Earth and Moon in two aspects: the difference in the ratio of the main components, i.e., silicon and iron, and the difference in the content of the volatile and high-melting elements. We will proceed from

the dynamics of the formation of the circumterrestrial swarm [11].

Sorting of the smallest particles with capture into the swarm and the related enrichment of the material of the Moon with silicates. Over the time of accumulation of the Earth and the evolution of the circumterrestrial swarm, the preplanetary particles and bodies passed /142 through repeated mutual collisions with velocities of several kilometers per second. It is natural to pose the question whether a systematic difference in the dimensions appeared between particles of different chemical composition and, consequently, of different physical and mechanical properties. Such a difference would first appear for metallic (iron) and silicate particles.

As Orowan [12] has shown, stony particles are brittle and give a large number of the smallest fragments as a result of collisions, whereas iron particles are ductile and plastic at rather low temperatures. They do not fracture in collisions, but merge, in contrast to stony particles. L.B. Al'tshuler and I.I. Sharipdzhanov have also noted a number of factors (strength, weldability), which stimulate the leading growth of iron particles in the preplanetary cloud [8]. Recent experiments on the collisions of metallic particles [13] have shown that in a wide interval of velocities - from 0.5 to $\tilde{\sim}$ 10 km/sec- metallic particles are welded together, whereas for velocities less than 0.5 km/sec, they experience a semi-elastic repulsion. Experiments with silicate particles [14] have verified the idea of Orowan that the behavior of silicate particles is completely opposite to the behavior of metallic particles, that is, in the interval of collision velocities 1.5 to 9.5 km/sec, the disintergration of the particles predominates over their adhesion. Only at velociteis less than 0.4 km/sec is their unification possible under the condition that there is some sort of cementing agent, for example, frozen volatile substances or electrostatic charge on the surface of the silicate particles. Investigations of the changes in material "processes" by impact with a velocity of several kilometers per second have shown that brittle silicate and hydride materials disintegrate into the smallest particles of micron and submicron dimensions [15], which are impossible to obtain by other methods of mechanical fractionation. Metals display only plastic deformation. The results of these experiments could be comparatively simply applied to the preplanetary particles, if they were strictly divided into silicates and iron. However, a significant part of the iron in the

preplanetary cloud could be oxidized and comprise particles in the form of FeO or Fe_3O_4. The oxides belong to the brittle, rather than to the plastic materials. There aren't any detailed experimental data for them on impacts with cosmic velocities. There is great interest in comparing the results of impact-processing of iron oxides and silicon oxides as the basic structural elements of the preplanetary material in the region of the terrestrial planets. There is an indication that at small velocities, the order of meters per second, the crushing of silica SiO_2 is greater than the crushing of magnetite Fe_3O_4 [16]. If the same behavior appeared also at cosmic velocities, then the dependence of the composition and size of particles could be represented in the form of a gradual increase in the iron content in the most massive bodies and silicates in the smallest fraction. In this case, a basis appears for the enrichment of the circumterrestrial swarm by silicate material due to the predominating capture of the small fraction. /143

Let us explain this assertion with a specific example. Let the distribution of the interplanetary particles and particles of the circumterrestrial swarm be described by exponential functions of the form $dN(a) \sim a^{-n}da$ with exponents n_1 and n_2 respectively, where $n_1 = 3.5$; $n_2 = 2n_1 - 3 = 4.0$ (cf. Chapter 4.2). We further assume that the density of the preplanetary particles and bodies, which characterizes the admixture of iron, smoothly increases with increasing size of the particles. We assume that the smallest particles of radius, say, 1μ have a density of $\delta(a_{min}) = 2.5$ gm/cm^3 (approximately the same as the lightest silicates without iron admixture), and the density of the largest bodies of radius 1000 km equals $\delta(a_{max}) = 4.1$ gm/cm^3 (approximately the same as the material of the Earth with the iron core at zero pressure and a total iron content of about 35%).

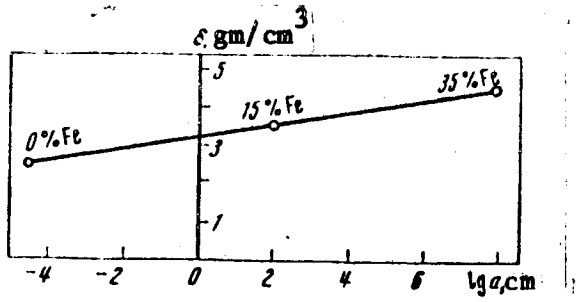

Figure 5.6. Hypothetical dependence of the composition of preplanetary bodies on their dimensions.

Assuming that this dependence of density on dimensions is linear-logarithmic, we obtain:

$$\delta(a) = \delta(a_{min})[1 + 1/6 \lg(a/a_{min})]. \quad (5.23)$$

It is represented in the form of a straight line joining the two selected

extreme values of δ as a function of lg a (Figure 5.6). Then one can easily show that the average density of the material for the ensemble of particles with exponents $n_1 = 3.5$, from which the Earth presumably grew, will be:

$$\bar{\delta}_1 = \delta(a_{min})[1 + 1/6 \lg(a_{max}/a_{min}) - 0{,}145] \approx 4{,}0 \quad gm/cm^3 \qquad (5.24)$$

i.e. the density and, consequently, the macrocomposition close to the present composition of the Earth. For the ensemble with the distribution with $n_2 = 4$, which is characteristic of the circumterrestrial swarm, we obtain the average density:

$$\bar{\delta}_2 = \delta(a_{min})[1 + 1/12 \lg(a_{max}/a_{min})] = 3{,}5 \quad gm/cm^3 \qquad (5.25)$$

i.e. a composition similar to the composition of the Moon in iron content.

In spite of the extreme simplification, this example clearly indicates the possibility of the fractionation of bodies by chemical composition, which results from the peculiarities of their growth with the difference in physical and mechanical properties of their component parts. Here we have been concerned only with the fractionation of metals and silicates in general. The possibility of fractionation between calcium and aluminum silicates on the one hand, and magnesium and iron on the other, is discussed in the work of Kaula [17] in connection with the enrichment of the material of the Moon by plagioclase, containing much CaO and Al_2O_3. There were undoubtedly other sorting varieties in the formation of the satellite systems of the other planets. /144

The partial loss of the volatile and semi-volatile elements in the circumterrestrial swarm and the related enrichment of the material of the swarm with high-melting components. The lunar basalts are relatively depleted as compared to the terrestrial basalts in many elements, whose common property is their volatility, and, conversely, enriched with high-melting elements (cf. Chapter 1). The basalts of the lunar maria are the products of magmatic differentiation of great depths of the Moon; thus, its composition reflects its global composition. The loss of volatile components of small atomic weight, in particular molecules of water, is possible from the surface of the Moon. But the deficit in the lunar basalts as compared to terrestrial basalts of the low-melting elements with atomic weights of about 200 can be

explained only by their loss by the material of the Moon in the process of its accumulation. This refers mainly to the elements Pb, Bi and Tl. The possibility of the deficit in the Moon of volatile elements results from the peculiarities in the formation of the circumterrestrial swarm. The active formation of the swarm must have occurred during the time when the mass of the growing Earth had reached approximately 2/5 - 2/3 of its present value. Thus, the circumterrestrial swarm is a secondary and later formation as compared to the Earth, and the material of the Moon must have been in a scattered state longer - at first in the preplanetary cloud, then in circumterrestrial orbits. This lag in accumulation is no less than $5 \cdot 10^7 - 10^8$ years. The small bodies and particles experienced over this time multiple collisions with velocities determined by the perturbations of the nucleus of the Earth: in the preplanetary cloud the peculiar velocities on the average of 3 - 4 km/sec, with capture into the swarm 5 - 7 km/sec.

The large impact energy (up to 10^{11} erg/gm) led to multiple fracturing of particles and the selective evaporation of material with the subsequent condensation of the evaporated atoms onto solid particles or being "blown away" under the effect of the solar wind. As Kuiper showed [18], for a velocity of solar protons of 1000 - 3000 km/sec the collision cross section of the protons for any atoms in interplanetary space is sufficiently large to ionize and eject them from the solar system. This "blowing away" must have mainly affected the most volatile substances (water, the low-melting elements Pb, Bi, Tl), since the high-melting substances were least subjected to selection. It is /145
natural to assume that those portions of the preplanetary material, which fell into the satellite swarm, were already somewhat more depleted in volatile compounds than those from which the nucleus of the Earth was formed. The sorting process also continued with the further simultaneous enrichment of the swarm and the growth of the Earth.

It can be shown that the preplanetary cloud in the radial direction from the Sun had an optical thickness no greater than unity right up to the orbit of the Earth, if the distribution of the preplanetary bodies followed the law $dN(a) \sim a^{-n_1}da$, where $n_1 \leq 3.5$ with a mass of the material equaling the mass of the terrestrial planets, and for

$a_{max} \approx 10^8$ cm. Consequently, at least at the periphery of the circumterrestrial swarm, one can also expect a sufficient transparency. In this region, each free atom of lead with a transverse cross section $\approx 10^{-15}$ cm^2, even with the present intensity of the solar wind of $2 \cdot 10^8$ protons/cm^2-sec, has the possibility of colliding with an energetic proton and flying from the swarm approximately once every two months. The condensation of the atom onto a particle for an optical thickness of the swarm $\tau \approx 1$ occurs after a time comparable to the time of revolution about the Earth, i.e., also several months at the periphery of the swarm. If the solar wind was much more intensive in the past than at present, then the ejection of the volatile elements from the outer part of the swarm must have been more effective than their condensation onto particles. The volatile elements are not selected with the accumulation of the Earth itself.

It should be expected that a majority of the elements were preserved in the inner, less transparent part of the swarm, since the density of the swarm and consequently, the nontransparency must have increased toward the center. One can assume, in particular, that significantly less water appeared in the initial composition of the Moon, than in the composition of the Earth. Anders assumes that the relative water content in the Moon is 50 times less as compared to the Earth [19]. This only strengthens the arguments expressed against the existence of a hydrosphere and atmosphere in the early history of the Moon with the assumption of the same relative water content in both bodies. The explanation of the deficit of the volatile components in the Moon by Anders and co-workers seems less plausible to us, since they assume the coincidence of the very short epoch of condensation of the low-melting elements at the very earliest stage of the preplanetary cloud (the first 10^5 - 10^6 years) with the epoch of completion of the growth of the Earth and Moon. The growth itself, according to present ideas, extended no less than 10^8 years, in favor of which the data of space chemistry also indicates [20]. Ringwood [21] explains the deficit of volatile compounds, as we do, by the solar wind. However, his model for the formation of the Moon from a primary thick iron-silicate atmosphere of the Earth is unfounded (too short a time scale for the formation of the Earth, unclear mechanism for the formation of a satellite from the atmosphere, etc.; cf. Chapter 3.2).

5.4. History of the Lunar Atmosphere

The problem of the loss of the volatile components of the lunar material before its accumulation closely adjoins the problem of the loss of vapors and gases from the lunar surface. The Moon at present has no atmosphere, and the lifetime for any very tenuous cloud of gases with atomic weight less than 40 (argon) does not exceed 10^8 years [22]. The heavier gases or vapors could have been held to the surface of the Moon by its gravitational force for a longer time. The absence on the Moon of even a thin envelope of the heaviest inert gases - krypton and xenon - speaks in favor of the blowing away of all traces of the lunar atmosphere by the solar wind, since in the opposite case such an envelope could have been accumulated over the time of existence of the Moon.

There is also interest in evaluating the maximum thickness of the lunar atmosphere in the past and also in determining the possibility of the existence of water in the liquid phase on the early surface of the Moon. It had already been shown in 1963 in work [23] that with the gradual degassing and defluidization of the lunar interior in the process of heating of the Moon by radiogenic heat, there could have been on the Moon in the past only a very thin atmosphere of water vapor, carbon dioxide and nitrogen - the main components of the degassing of the interior of the terrestrial planets. The remaining components of the atmosphere were in negligibly small quantities. It was also established that there could never have been water in the liquid phase, if it were released from the interior of the Moon approximately the same as on the Earth. Direct investigations of the chemical composition of lunar rocks, begun in 1969, have shown that there are no sedimentary rocks on the surface of the Moon, no hydrated silicates, no traces of water erosion, no frozen water. Thus, our calculations have been verified. At the same time, the theoretical predictions of Gilvarry [24] on the existence of seas of water on the Moon in the past and Urey [25] on the existence of lunar rivers in the past have fallen. The initial content of volatile components in the Moon is now estimated as much lower than on the Earth. There is no direct data for water, but according to the ratio found in lunar samples for the volatile thallium to the nonvolatile uranium ($(Tl/U)_{Moon} = 2 \cdot 10^{-4} (Tl/U)_{cosm}$),

Anders and others [19] estimate the primary cosmic amount of water in the Moon as equivalent to a layer on the surface of thickness at most 22 m. Meanwhile, the "terrestrial" content would give a layer of thickness 1 km, and it was for this larger value that we reached the negative conclusion in 1963 about the existence on the Moon of a hydrosphere and dense atmosphere in the past. One can now assert with even greater assurance that there was never liquid water on the surface of the Moon, and the search for permafrost under the surface would hardly be successful.

We will present here an estimate of the density of the atmosphere of the Moon during the epoch of degassing of the interior according to [23]. It is natural to assume that the period of active degassing of /147 the interior of the Moon lasted approximately as long as the epoch of the great lava flows, i.e. about one billion years. This length of the epoch of maximum melting of the interior of the Moon is obtained from the data on its thermal evolution.

We assume that the composition of gases enriching the lunar atmosphere was similar to that of the Earth. The data usually taken for the Earth is that of Ruby [26], according to whom about 330 kg of H_2O, 18 kg of CO_2, 6 kg of Cl_2, 1 kg of N_2, 0.5 kg of S and 0.2 kg of H_2 were released from the interior per square centimeter of surface over the entire history of the Earth. For the same initial concentration of volatile substances in the Moon and for the same degree of differentiation of its interior there must have been released, to 1 cm^2 of its surface, 6 times smaller amount of the volatile components. Actually, the volatile compounds released were 10 - 20 times less, since the lunar material from the very beginning was poorer in volatile substances than the Earth [19]. The total amount of water on the lunar surface was evidently little more than 2 kg/cm^2, CO_2 - 100 gm/cm^2, and nitrogen, sulfur and hydrogen were even significantly less. By taking into account that there was little chlorine and that because of its great chemical activity it could combine with rocks, it can be neglected in considering the lunar atmosphere.

The uniform release from the interior during the course of 10^9 years of the amount of water and carbon dioxide assumed above determines

the flux of volatile compounds F_0 through the lunar surface into the atmosphere. The accumulation of the atmosphere is accompanied by an increase in the flux of molecules dissipating from it. The thickness of the atmosphere can grow only until the flux from the interior is comparable to the flux lost into space. The time for establishing such an equilibrium is very small, the order of 10^8 sec. The density of the atmosphere calculated for such an "equilibrium" state during the degassing period should be considered the maximum in the history of the Moon, since with a decrease in the emission of volatile components from the interior the atmosphere must have dissipated and reached its present extremely tenuous state.

The estimate of the loss rate of the atmosphere is usually carried out with the well-known formula of Jeans for dissipation, giving the number of particles L lost by the entire atmosphere per second:

$$L = 4\pi R_1^2 \frac{c}{\sqrt{6\pi}} n_1 e^{-Y_1}(1 + Y_1), \tag{5.26}$$

where

$$Y = \frac{GM_\zeta \mu}{\Re T R}, \quad c = \sqrt{\frac{3\Re T}{\mu}}, \tag{5.27}$$

M_ζ is the mass of the Moon, R is the distance from its center, $\Re$ is the gas content, T is the temperature of the atmosphere, μ is the molecular weight, n is the number of particles per cm^3. The index 1 indicates that the corresponding quantities refer to the level from which the dissipation occurs. If the atmosphere consisted of one type of molecule, then, by equating the quantity $L/4\pi R_0^2$ (where R_0 is the radius of the Moon) to the flux F_0 of particles released from the interior, one could find from Formula (5.26) the concentration of particles n_1 at the dissipation level and proceed from it to the concentration n_0 at the surface, which is particularly simple to accomplish in the case of an isothermal atmosphere, for which the barometric formula gives:

$$n_1 e^{-Y_1} = n_0 e^{-Y_0}. \tag{5.28}$$

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In the case of a multi-component atmosphere, such a calculation gives the correct order of magnitude of the density n_0 only for the most abundant component - water vapor: $n_0 \gtrsim 10^{12}$ cm^3 for T = 400°K. The condition for the dissipation level R_1:

$$\tau = \int_{R_1}^{\infty} n\sigma dR \leqslant 1, \qquad (5.29)$$

where σ is the effective cross section of the molecules, τ is the optical thickness. Condition (5.29) indicates that at the dissipation level the molecular mean free path must be the order of the radius of the planet.

The Moon has a comparatively small mass, and thus the barometric formula gives a rather weak decline in the density of the atmosphere with altitude, while for some n_0 greater than a specified limit, one can describe cases when the theoretical dissipation level is either very far from the surface or is not reached at all. However, this does not imply that the dissipation of the atmosphere becomes ineffective. Actually, the dissipation level must be determined by the real physical boundary of the atmosphere R_2. It is obvious first of all that the atmosphere cannot extend further than the level corresponding to Y = 3/2, at which the mean thermal velocity of molecules equals the parabolic velocity and, consequently, the molecules are no longer held by the gravity of the Moon. For molecules of H_2O at T = 400°K, the boundary of the atmosphere defined in this way is $R_2 \approx 10\ R_0$. However, because of the effect of the "solar wind," which blows away the atmosphere more effectively the greater its size ($\sim R_2^2$), the actual boundary of the atmosphere is significantly closer.

We assume that at a distance of 1 AU from the Sun the corpuscular flux (protons) with an average velocity of v_{ave} = 200 km/sec is 10^9 particles/cm^2-sec. Since 10^{10} - 10^{11} molecules/cm^2-sec are released from the interior of the Moon, then, by taking the cross section of only the Moon itself πR_0^2, we find that there are 40 - 4000 molecules released from inside per particle of the "solar wind" incident on the Moon. Even having very high energies, the particles cannot remove by collisions such a large number of molecules from the surface of the Moon, especially since the momentum of the particles is directed toward the Moon and only a small part of the molecules, with which they collide, can obtain a momentum directed outward from the Moon. But if the atmosphere has a dimension $R_2 \gg R_0$, then the effectiveness of its blowing away is significantly greater. This also determines the outer boundary of the atmosphere. /149

An approximate estimate of the location of the boundary can be made in the following manner. We will assume that the effective blowing away of the atmosphere is produced by particles flying tangent to the lunar surface at the distance R_2 in a band of width ΔR_2, where each proton encounters just as many molecules as it can eject into space. The momentum delivered by the protons to the atmosphere is $m_H\, 2\pi R_2 \Delta R_2 v_{ave} \cdot 10^9$ per second. It must equal the mass of the gases $4\pi R_0^2 \mu m_H F_0$ released from the interior of the Moon per second, multiplied by the escape velocity at the distance R_2, equaling $\sqrt{2GM/R_2}$. Taking $F_0 = 10^{11}$ cm$^{-2}\cdot$sec^{-1}, $\mu = 18$, we find:

$$R_2 \Delta R_2 = 1{,}8 \cdot 10^{-4} R_0^2 \sqrt{2GM/R_2}. \qquad (5.30)$$

The quantity ΔR_2 must be the order of the altitude of the homogeneous atmosphere H. Since the surface density (the total number of molecules in the path of the particles per cm^2) decreases with R more gradually than the volume density, then ΔR_2 equals at least 2H:

$$\Delta R_2 \approx 2H = \frac{2R_2}{Y_2} = \frac{2R_2^2}{Y_0 R_0}.$$

Thus, we have:

$$(R_2/R_0)^{3/2} \approx 21{,}6 Y_0. \qquad (5.31)$$

For $\mu = 18$ at $T = 400°K$, $Y_0 = 15.2$ and $R \approx 5R_0$, and at $T = 1000°K$, $Y_0 = 6.1$ and $R_r = 3.5\ R_0$.

The blowing away of the upper part of the atmosphere by the "solar wind" is a supplementary mechanism in the loss of gases, since it is accompanied by the expansion of the entire atmosphere into the vacuum. Thus, the rigorous solution of the problem of the dissipation of the lunar atmosphere requires the development of a theory of the loss of gases of a nonsteady-state expanding atmosphere. Since there is no such theory, and we are interested in the maximum estimate of the density of the atmosphere, we will consider the effect of only the thermal dissipation of gases, assigning to the particle blowing the role of only a factor limiting the dimension of the atmosphere. In accordance with this, we take the following model of the lunar atmosphere:

1. We take $R_2 = 5R_0$ at $T = 400°K$ and $R_2 = 3.5\ R$ at $T = 1000°K$ as the boundary of the multi-component atmosphere.

2. Because of the sharp difference in temperatures on the day and night sides, the atmosphere is mixed convectively and the relative

concentration of gases is the same at all altitudes.

3. The density distribution up to the level R_2 corresponds to the barometric formula for the isothermal atmosphere with an average molecular weight $\bar{\mu}$. /150

4. The dissipation level R_1 is determined by Condition (5.29):

$$\sigma \int_{R_1}^{R_2} n(R)\,dR = \sigma n_1 \int_{R_1}^{R_2} e^{Y-Y_1}\,dR = 1, \tag{5.32}$$

where σ is the effective molecular collision cross section ($\sigma \sim 10^{-15}$). Eliminating n_1 with the help of (5.26) - (5.28) and expressing L in terms of the flux F_0, we find the relation for Y_1:

$$\frac{e^{Y_1}}{Y_1} - EiY_1 + B\frac{1+Y_1}{Y_1^2} = \frac{e^{Y_2}}{Y_2} - EiY_2, \tag{5.33}$$

where

$$B = -\frac{c}{\sqrt{6\pi}}\frac{Y_0}{\sigma R_0 F_0}, \qquad EiY = \mathrm{li}\,e^Y = \int_0^{e^Y} \frac{dt}{\ln t};$$

R_1 is determined, according to (5.27), by Y_1. We take the value of R_1 found for the most abundant component for all the gases. At T = 400°K, $R_1 = 3.33\ R_0$ and at T = 1000°K, $R_1 = 1.68\ R_0$.

5. The amount of dissipating gases equals the amount of gases released from the interior:

$$L = 4\pi R_0^2 F_0. \tag{5.34}$$

6. Since a larger part of the lunar atmosphere is accessible for irradiation by the ultraviolet radiation of the Sun, CO_2 and H_2O are absent in the upper part of the atmosphere and, in particular, at the dissipation level. In place of them, there are the corresponding number of CO, O and H_2 molecules, which also determine the dissipation rate.

It should be emphasized that the assumption about dissociation significantly affects the results, since CO_2 dissipates much more gradually than CO. At the same time, the dissociation of H_2O accelerates the dissipation by only 2 times. Since the degree of dissociation of H_2O and CO_2 is unknown at the lunar surface itself, our calculations of n_0 is performed for two extreme versions: the case of complete

dissociation and the case when there is no dissociation.

The values of Y_1 are found for each gas separately from the assumed values of R_1, and the concentrations n_1 and average molecular weight $\bar{\mu}$ at the dissipation level are calculated with the help of relations (5.26) and (5.34). Then the transition is made to the concentration n_0 at the surface:

$$n_0 = n_1 e^{\bar{Y}_0 - \bar{Y}_1}. \tag{5.35}$$

from the barometric formula for $\bar{\mu}$. In the second version $\bar{\mu}$ increases approaching the surface. Since this change does not strongly affect the estimate of n_0, one can take without large error $\bar{\mu}$ as a constant equaling the average of its values at the surface and at the level R_1. The results of the calculations are presented in Table 8. /151

At a temperature of 400°K, the density of the lunar atmosphere is the order of $10^{11} - 10^{12}$ cm^{-3}, and at 1000°K — at most the order of 10^8 molecules per cm^3. However, for densities of $\lesssim 10^8 cm^{-3}$, the mean free path becomes large and the molecules often collide with the surface of the Moon, assuming its temperature. Although high temperatures are natural in a tenuous atmosphere at great distances from the surface, the cooling role of the surface in this case is great. Thus, values are also presented for the case when T increases with R according to a power law, changing from 400°K at the surface to 1000°K at the dissipation level.

Since the maximum density presented in the table corresponds to a density in the terrestrial atmosphere at an altitude the order of 150 km and higher, the first version should evidently be preferred. As was already noted, the calculation is based on the assumption that thermal dissipation is the only mechanism for the loss of gases and thus gives an upper limit of the density of the lunar atmosphere. If the roles of thermal dissipation and the solar wind were comparable in the removal of the lunar atmosphere, then the actual values of n_0 must be approximately two times lower than those presented in the table. Thus, one can assume that the density of the lunar atmosphere during the epoch of active release of gases from the lunar interior was $10^{10} - 10^{11}$ atoms/cm^3, i.e. a billion times less than the density of the atmosphere at the surface of the Earth.

TABLE 8

DENSITY AND COMPOSITION OF THE EARLY ATMOSPHERE OF THE MOON

Version	O_2		CO_2		N_2		H_2	
	Molecules /cm^3	% by weight	Molecules /cm^3	% by weight	Molecules /cm^3	% by weight	Molecules /cm^3	%by weight
			400° K					
I	$4\cdot10^{11}$	60	$1{,}5\cdot10^{11}$	37	$1{,}1\cdot10^{10}$	2,7	$1{,}4\cdot10^{10}$	0,3
II	$2\cdot10^{12}$	38	$1{,}1\cdot10^{12}$	57	$8\cdot10^{10}$	2,7	$1\cdot10^{11}$	2,2
			1000° K					
I	$8\cdot10^{7}$	74	$1{,}5\cdot10^{7}$	24	$1{,}1\cdot10^{6}$	1,8	$5\cdot10^{6}$	0,6
II	$8\cdot10^{7}$	55	$2\cdot10^{7}$	38	$1{,}5\cdot10^{6}$	1,8	$7\cdot10^{6}$	5,7
			400—1000° K					
I	$1{,}0\cdot10^{9}$		$2\cdot10^{8}$		$1{,}5\cdot10^{7}$		$7\cdot10^{7}$	
II	$1{,}1\cdot10^{9}$		$3\cdot10^{8}$		$2\cdot10^{7}$		$9\cdot10^{7}$	

At a temperature of 400°K, the density of saturated water vapor is about 10^{14} molecules/cm^3, i.e. four orders of magnitude greater /152
than the maximum value presented in the table. Hence, it follows that there could have been no liquid water on the Moon during the release of water from the interior over more than 10^5 years. This result is in clear contradiction to the conclusion of Gilvarry [24] on the existence of oceans on the Moon in the past over the course of billions of years, which was also reached on the basis of calculations of the rate of thermal dissipation of water vapor. Gilvarry takes 79 kg/cm^2 as the amount of water released from the lunar interior and assumes that it was all released over a short time. However, the main reasons for the divergence is not this, but the extremely long times for the loss of water by the Moon, which was found by Gilvarry: about 30 billion years at T = 400°L and about 3 billion years at T = 2000 - 3000°K. This result was obtained because of internal inconsistencies in the model of the atmosphere assumed by Gilvarry. Its dissipation level is located at an altitude of 2300 km above the surface of the Moon. This corresponds to a density at the surface at 400°K, 3 orders of magnitude lower than the density of the saturated vapor. Since the actual density must be close to the latter, then the time for the loss of water by the Moon was overestimated by approximately the same 3 orders of magnitude.

We see that there were never conditions on the Moon for the existence of a hydrosphere or any significant atmosphere. Thus, it is extremely unlikely that complex organic compounds like amino acids were synthesized, as postulated by Sagan in his early works [27].

It has been shown in this chapter that the participation in the formation of the Moon of large bodies comparable in mass results naturally from the dynamic peculiarities of the satellite swarm. Moreover, such a picture of the formation agrees better with the necessity of an initial high temperature of the Moon and the large-scale inhomogeneities in its interior than the formation directly from small bodies and particles.

Furthermore, the explanation of the differences in the chemical composition of the Moon and Earth within the framework of this model can be related to fractionation processes in the small component of the preplanetary material and the material of the satellite swarm at the stage preceding the formation of the large bodies. The satellite swarm was enriched with the small fraction, which because of collisions was depleted in iron and metals and also the volatile components, and instead of this was enriched with silicates, mainly aluminum and calcium silicates and the high-milting micro-elements.

The problem of the deficit of volatile elements in the Moon adjoins the problem on the loss of water and atmospheric gases from the surface of the Moon in the course of its evolution. By taking one billion years as a characteristic time for degassing and defluidization of the interior and the "terrestrial" content of water and gases, which is evidently greatly overestimated for the Moon, we have obtained that the lunar atmosphere could never have reached densities above $10^{-8} - 10^{-9}$ of that of the Earth and that there never was water in the liquid phase on the surface of the Moon. This conclusion completely agrees with the facts.

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Chapter 6

FORMATION OF THE SATELLITES OF THE OTHER PLANETS OF THE SOLAR SYSTEM

Until now we have considered the formation and evolution of the satellite swarm only in connection with the origin of the Moon in orbit around the Earth. An analogous process must have occurred with each growing planet, and our model of the formation of the Moon must be also applicable to the formation of other natural satellites of planets. /154

In recent years it has been learned in connection with progress in studying the internal structure and composition of the planets that there are not two, but at least three groups of large planets in the solar system: 1) the terrestrial group consisting mainly of heavy oxides and depleted not only in the gases H and He, but also in the group C, N, O according to their cosmic abundance; 2) the Jovian planets Jupiter and Saturn, in whose composition the mixture of H and He predominates; 3) the planets Uranus and Neptune, in whose composition the compounds of hydrogen with elements of the group C, N, O - mainly water in the liquid state - appear [1]. As yet there are no data about the composition of Pluto.

The planets of the first and third groups accumulated the preplanetary material primarily in the form of solid particles, the main flux of material for Jupiter and Saturn was due to the accretion of gas. The diversity of conditions of growth of the planets themselves reflected on the conditions for the formation of the satellite systems. We will consider here two types of satellite swarms: for planets without gas accretion and for planets with accretion.

The theory of gas accretion by Jupiter and Saturn is itself not yet sufficiently developed, although the general nature of the process has been described in quantitative form. The pattern of formation of the planets Uranus and Neptune is not yet clear, since much material was blown out of the solar system from the region of these planets.

Thus, our consideration of the formation of satellites should be considered preliminary.

We will also discuss the evolution of satellite systems. There is reason to think that some satellites fell to the planets, after approaching them because of tidal drag. Others changed their orbits such that commensurability of the periods of revolution was established. /155 We will also evaluate the maximum effect of tidal friction on the orbital inclination of satellites to the equator of the planets and, finally, we will present information about the parameter Q for the Jovian planets.

6.1. History of the Satellite Swarms of the Terrestrial Planets

The four terrestrial planets - Mercury, Venus, the Earth and Mars have at present three satellites in all: the Moon with a mass of 1/81 of the mass of the Earth, Phobos and Deimos with a total mass of $2 \cdot 10^{-8}$ of the mass of Mars, which in turn is only 10 times smaller than the Earth. Mercury, which is 2 times smaller in mass than Mars, has no satellites. Venus also has no satellites, although its mass is close to that of the Earth and exceeds the mass of Mars by 8 times. It seems impossible to devise a single theory, which would explain such a great difference in the present ratios of the masses of satellites to the mass of the planets. However, one can try to take into account the evolution of satellite systems of the terrestrial planets and to reconstruct the picture, which was produced soon after the formation of the planets. The basic factors of the evolution were evidently tidal friction in the systems of Mercury and Venus and the decisive role of external impacts in the system of Mars.

The disappearance of the satellites of the slowly rotating planets Mercury and Venus. The rotation of Mercury and Venus around their axes occurs so slowly that the specific momenta of both planets deviates significantly from the empirical straight line connecting the values of momenta of all the other planets (cf. Figure 3.3). The closeness of these planets to the Sun and also the synchronization of the frequency of the rotation of Mercury with the angular frequency of its

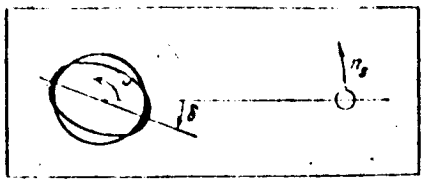

Figure 6.1. Tidal interaction of a satellite with a slowly rotating planet.

motion near perihelion, where the tidal friction is particularly strong [2] undoubtedly indicates the leading role of tidal friction in slowing the rotation of these planets. Evidently, the dissipation parameter Q for Mercury and Venus is no greater than 100. It is possible that both planets were slowed down very long ago at the beginning of the existence of the solar system (at least, Mercury). In this case, one can try to find out what satellite system a slowly rotating planet will have. Such an investigation was performed recently by J. Burns during his work at the IFZ AN SSSR [3], and independently shortly after by Ward and Reid [4]. Our further discussion of this problem is quoted from [3].

We will consider only direct satellites. For satellites with retrograde revolution, the effect of the fall to a planet with any angular velocity of its rotation is obvious, but with the condition of a sufficiently large dissipation of tidal energy in the interior of the planet [5]. The sign of the difference $(\Omega - \omega)$ is important in the case of direct satellites. Mercury now has direct rotation with a period of 58,663 ± 0.021 days, Venus has retrograde rotation with a period of 243.09 ± 0.18 days [3]. Direct satellites in circular orbits within the entire sphere of influence for both planets must revolve more rapidly than the surface of the planet rotates, i.e. $\Omega - \omega < 0$. In this case, a lead of the tidal bulge with respect to the culmination of the satellite is observed on the planet, and not a lag (Figure 6.1): the satellite is slowed, and its orbit contracts toward the planet at an ever increasing rate. The final result can be only the destruction within the Roche limit and fall to the planet. The equation for the change in the semimajor axis of the orbit of a satellite m will be:

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$$\frac{da}{dt} = 3k_2 G^{1/2} M^{-1/2} m a^{-11/2} R^5 \sin 2\delta = -\tfrac{2}{3}(GM\Omega^{-5})^{1/3}\frac{d\Omega}{dt}. \qquad (6.1)$$

The time of contraction of the satellite orbit from the radius $l = a/R$ to the surface of the planet is:

$$\Delta t = \tfrac{1}{13}(3G\pi\rho)^{-1/2}(Q/k_2)(M/m)\,l^{13/2}; \qquad (6.2)$$

It is smaller, the larger the mass of the satellite and the closer it is located to the planet at formation. Figure 6.2 shows the dependence

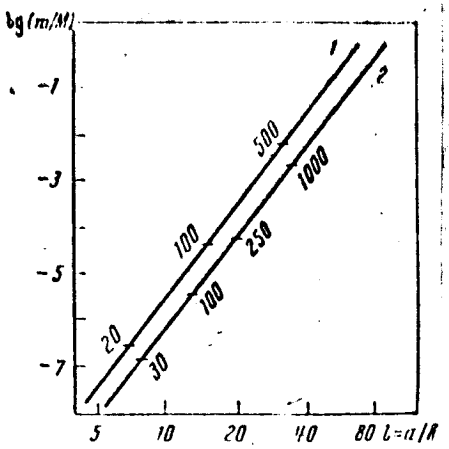

Figure 6.2. Critical relations of the mass and orbital radius for satellites of Mercury (1) and Venus (2): digits on the curves denote the dimensions of satellites in km.

Figure 6.3. Photographs of Phobos (a) and Deimos (b), obtained from Mariner-9.

of mass and distance for satellites of Venus and Mercury, which separates the region of disappearing satellites and the region of preserved satellites after a time Δt. It was assumed that $Q = 100$; $k_2 = 0.05$ for Mercury and 0.25 for Venus. The time Δt was taken as half the age of the solar system, that is $2.3 \cdot 10^9$ years.

The latter assumption is equivalent to the hypothesis that the slowing of the planets themselves by the solar tides to their present slow rotation also lasted no more than half the time of existence of the solar system, i.e. $2.3 \cdot 10^9$ years. The meaning of Figure 6.2 is such that all satellites, for which the masses and distances occur to the left and above the straight lines 1 and 2, must have had time to fall to the planet. We see that satellites with radii no less than 1000 km for Venus and 500 km for Mercury could have fallen to the planet from a distance of about 40 times the planetary radius. Smaller satellites could have fallen accordingly only from closer orbits. The present slow retrograde rotation of Venus must be related to the fall to its surface of a small retrograde satellite [5]. One can conclude that Venus and Mercury in the past could have had significant satellite swarms and then satellite system, which later disappeared as a result of tidal evolution. The estimates presented above for the masses of disappearing satellites should be considered preliminary, since until now neither the present values of Q for Venus and Mercury nor the

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history of their tidal braking by the Sun are known. We will turn again to the problem of the probable masses and momenta of the satellite swarms of Mercury and Venus below.

Destruction of the small satellites of Mars by impacts of interplanetary bodies. The dimensions of the satellites of Mars are small. If they are represented in the form of triaxial ellipsoids, then the semiaxes of Phobos will be 14 ± 1; 11.5 ± 1; 10 ± 1 km, and for Deimos 8 ± 2; 6 ± 1; 5 ± 1 km with consideration of the irregularity of their shape. Photographs of Phobos and Deimos, obtained from space (first by Mariner-9 in 1971), are clear illustrations of the destruction of small solid satellites by impacts of interplanetary bodies (cf. Figure 6.3). The impact craters on Phobos and Deimos appears as holes without ridges. This implies that all the rock ejected by the impact left the satellite, which is not surprising, since the escape velocity at the surface of the satellites is at most 10 - 20 m/sec. The craters comparable in size to the satellites themselves and also the irregular shape of the latter indicate that they could be only fragments of larger bodies. Consideration of the present bombardment in the region of the orbit of /158
Mars indicates that the satellites of Mars at present experience surface erosion, but the extrapolation of this erosion into the past leads to "initial" dimensions of Phobos and Deimos not greatly exceeding their present dimensions [6]. At the same time, consideration of the much more intensive bombardment at the concluding stage of growth of the planets gives a significantly greater erosion for the past and even fragmentation of bodies of asteroid dimensions. The circumstances for collisions in the vicinity of Mars must have resembled the circumstances close to the growing Earth, which we consider in Chapter 4.6. The acceleration of bodies in the field of Mars equaling $\sqrt{2GM_M/l}$, where l is expressed in units of its radius, was approximately 2 times lower than for the Earth at the same distance, but the velocity "at infinity" could have been larger than in the zone of the Earth because of perturbations from Jupiter. Thus, it has not been excluded that the original dimensions of Phobos and Deimos were the order of hundreds and not tens of kilometers. It is possible that the initial distances of the satellites of Mars were greater than the present distances. Lyttleton has noted that for bombarding particles with a low planet-centered momentum the satellite being disrupted, besides losing mass, also loses momentum

and approaches the planet [7]. With a significant loss of mass, this effect could be larger than the tidal effect. The present effect of tidal friction on the size of the orbits of Phobos and Deimos is evidently small because of their small mass. Phobos is now approaching Mars, while Deimos is moving away, but the absolute rate of these changes is insignificant.

Thus, the initial masses of the satellites of Mars could have been 2 or 3 orders of magnitude greater than the present masses; the dimensions of their orbits could also have been greater. It should be noted that if the radii of the satellites of Mars had once reached $\tilde{\sim}$ 400, then the destruction would have been ineffective and the large satellites would have remained intact up to the present. Thus, the radius of $\tilde{\sim}$ 400 can probably be considered as the upper limit for the satellites of Mars.

Mass and planet-centered momentum of the satellite swarms of the terrestrial planets. Let us now apply the considerations on the formation of the terrestrial satellite swarm, which was developed in the fourth chapter, to the other terrestrial planets.

The growth of the planets is described by Formula (4.6), the growth of the satellite swarm by Formula (4.45). By neglecting the contribution of the material of the swarm in the mass of the planet as compared to its direct growth and by integrating separately the terms with μ_1 and μ_2, we have:

$$\mu_1(M) \cong D_1 (M^2 - m_0^2)/2,$$
$$\mu_2(M) \approx \mu_0 \exp[3D_2 (M^{1/3} - m_0^{1/3})], \qquad (6.3)$$

where, as before, μ_1 denotes the contribution of "free-free" collisions and μ_2 the contribution of "free-bound" collisions; M is the present mass of the terrestrial planet, and the coefficients D_1 and D_2 have a form analogous to (4.51) and (4.52), but with consideration of the numerical corrections for the zones of the other planets, i.e. other dimensions of the zones σ, the effective particle size and the dimensions of the sphere of influence of the planet. /159

We assume for simplicity that all these parameters for the terrestrial planets were the same. Then one can reach the conclusion that:

$$\mu_1 \sim M^2 \text{ (neglecting } m_0^2\text{)},$$
$$\mu_2 \sim \mu_0 \exp[\text{const}\,(R - r_0)], \qquad (6.4)$$

where r_0 is the initial radius of the planet, to which μ_0 refers, and R is the final radius of the planet. It is seen from relations (6.4) that the dependence of the mass of the satellite swarm on the mass of the planet cannot be represented as linear in any case. The contribution of "free-free" collisions μ_1 increases as M^2, i.e. μ_1 for Mars and Mercury is 100 - 400 times, and for Venus 1.5 times smaller than for the Earth. We have seen that the absolute contribution of μ_1 is very small even for the Earth. Thus, it is smaller for the other planets. The value of μ_2 depends on μ_0 - the initial mass of the swarm produced by "free-free" collisions and on an exponential factor, which can behave differently. If, for example, the characteristic time over which the mass of the swarm reaches the mass of the free particles contained in the sphere of influence of the planet is large, then it can be shown that $r_0 \approx R$, i.e. over the time of growth of the planet from r_0 to R the swarm did not reach the stage of rapid enrichment due to the sticking together of previously free particles and remains tenuous. A massive swarm, which could not have been produced without the "exponential" growth, was necessary for the formation of the Moon.

We assume that there was the same growth for the swarms of the other planets, but that the total mass of the swarm was proportional to μ_1. Although this is a somewhat artificial assumption, we can obtain with its help a rough upper limit for the mass of the satellite swarms μ and the equivalent radius of a single satellite a_{eq}, which equal:

	Mercury	Venus	Earth	Mars
μ/μ_ζ	$2{,}5 \cdot 10^{-3}$	0,6	1,0	0,01
a_{eq}, км	250	1500	1738	400

Let us now evaluate the planet-centered momentum of the hypothetical satellite swarms. We assume, as in Chapter 4, that it equals $I\omega_c$. For the same concentration exponent of the material of the swarm, the specific momentum $k = l_1^2\omega_c$. Since:

$$l_1^2 = (R\sqrt[3]{M/M_\odot})^2, \quad \omega_c = \sqrt{GM_\odot/R^3},$$

then:

$$k \sim \mathfrak{R}^{1/2} M_p^{1/2}, \tag{6.5}$$

where $\mathfrak{R}$ is the distance from the Sun, and the equivalent initial distance of a circular satellite from its planet in units of its radius /160 $l_0 = a_0/R$ can be found from the relation:

$$k = \sqrt{GMa_0} = \sqrt{l_0}\sqrt{GMR} \sim \sqrt{l_0} M^{1/2},$$

i.e.:

$$l_0 \sim k^2/M^{1/3} \sim \mathfrak{R}. \tag{6.6}$$

We take l_0 as $20\,R_\odot$ for the Moon. We obtain the equivalent initial distance of direct satellites for the different planets equaling $8\,R_\mathrm{☿}$ for Mercury, $14\,R_\mathrm{♀}$ for Venus, $20\,R_\odot$ for the Earth and $30\,R_\mathrm{♂}$ for Mars.

Thus, if one carries out a complete analogy to the Earth-Moon system (which is justified in this case in the absence of a better approach), then we obtain for Mercury and Venus satellites with a radius of 250 and 1500 km respectively with orbits removed by 8 and 14 times the planetary radii. Such satellites must have already disappeared in falling to the planets, if the assumptions, on which the graph in Figure 6.2 is based, are true. We have found for Mars the possibility of having at the end of growth, satellites with a radius no greater than 400 km at a distance of $30\,R_\mathrm{♂}$, i.e. significantly more remote than Phobos and Deimos (their present distances are 2.76 and $6.92\,R_\mathrm{♂}$ respectively). It is encouraging that we have obtained an upper estimate for the radius of a satellite of Mars by two different ways: first from the considerations that a satellite of such a dimension would probably have been little disrupted and remained intact up to the present in the form of an asteroid body, and then from the analogy of the formation of the swarms of Mars and Earth, although the estimate here is less convincing. Thus, one can think that the present satellites of Mars are remnants of larger bodies, which were formed about Mars in more remote orbits.

Let us evaluate the maximum contribution of the hypothetical satellite systems of the planets Mercury and Venus to the rotation of these planets with their fall to the planet. This rotation is in addition to that which the planet acquired during its formation and could have lost due to solar tides until the approach of the satellites began with the transfer of their orbital momentum. We will make use of the ideas developed in Chapter 4.5.

TABLE 9

VALUES OF THE TOTAL MOMENTUM K BROUGHT TO THE PLANETS BY THE SATELLITE SYSTEMS

Planet	μ/μ_ζ	k/k_ζ	K/K_ζ
Mercury	$2{,}5\cdot10^{-3}$	0,14	$+3{,}5\cdot10^{-4}$
Venus	0,6	0,73	+0,44

We have specific momentum of the material of the satellites /161
$k \sim \mathfrak{M}^{1/3}M^{1/3}$; let the mass of the satellites $\mu \sim M^2$, then we obtain the following values for the total momentum $K = k\mu$ (in units of the initial momentum of the Moon) (Table 9).

We see that the total momentum for Mercury would have been only a thousandth of that of the Earth, and for Venus less than half that of the Earth. Then the equivalent angular rotational velocity of the different planets Ω can be found from the relation:

$$K = I\Omega = MR^2,\ \Omega \sim M^{1/3}\Omega,\ \Omega \sim K/M^{1/3},$$

i.e. in fractions of the terrestrial angular velocity:

$$\Omega_M = 0{,}065\Omega_E,\ \Omega_V = 0{,}7\Omega_E. \qquad (6.7)$$

Here Ω_E is the angular velocity of the Earth, which it would have acquired only die to the fall of satellites, i.e. if it had obtained a momentum $K + K_\mu$ (cf. Table 6).

By taking $K + K_\mu = 2 \cdot 10^{41}$ CGS, which corresponds to the rotation of the Earth with a period of 6 or 7 hours, we obtain an equivalent period of rotation of the planets due to "falling satellites" for Mercury of $\tilde{\sim}$ hours and for Venus $\tilde{\sim}$ 10 hours, which does not fall outside the limits of the initial periods of rotation of the planets, which are not less than several hours. By assuming that the initial rotation of Mercury and Venus could have been slowed by solar tides to synchronization with the period of revolution or almost to synchronization over half the lifetime of the solar system, it is logical to assume that over the second half of the lifetime Venus and Mercury are able to lose the same rotation. The present rotation of Mercury completely corresponds to synchronization [8], and Venus is not quite synchronized. It has an axial momentum, comparatively small in absolute value and negative in sign $K_V \cong\approx -2 \cdot 10^{38}$ CGS $\tilde{\sim} - 4 \cdot 10^{-3} K_E$. It is possible that this

momentum was communicated to it by a small retrograde satellite falling significantly later, because retrograde satellites were usually located at the periphery of the satellite systems. There could have been a satellite with a mass of about $0{,}001\mu_{\zeta}$ moving in a retrograde orbit at (18 - 20) R_V when the rotation of Venus was synchronized to the orbital period (direct rotation with a period of 225 days). For a larger mass of the retrograde satellite, the orbital radius must be smaller.

But the cause of the appearance of the retrograde rotation of Venus from an almost synchronized rotation could have been related to solar tides in the atmosphere, for which a lead is possible [9]. This theory is not yet developed. With the refinement of knowledge about the physics of the atmosphere of Venus and also about the parameter Q for its interior, a more complete representation about the history of its rotation can be compiled. One can expect that Q for Venus is small as compared with Q for the Earth. This is related to the fact that the aesthenospheric layer of Venus can be found closer to the surface (the difference in the boundary temperatures for the Earth and Venus is about 300°). The change in content of radioactive elements at the surface of Venus with the help of the Venera-8 spacecraft have high values typical of terrestrial granites [10]. This indicates the active thermal history of Venus, the present hot state of its interior and small values of Q. /162

Thus, the history of the satellite swarms of the terrestrial planets, considered according to the type of formation of the circumterrestrial swarm, does not contradict present facts.

6.2. History of the Satellite Systems of the Jovian Planets.

The Jovian planets - Jupiter, Saturn, Uranus, Neptune - have developed satellite systems. The various regularities of these systems are well known. One of these is the similarity of the law of satellite distances in the system of a given planet to the law of planetary distances from the Sun. Its main feature is the progressive increase in distances between the more remote satellites. The origin of such a law should be sought in the expansion of the supply zones of the growing bodies with distance from the central body (cf. Chapter 5.1). However,

besides the progressive increase in the intervals between the satellite orbits, their periods of revolution are rather often in simple ratios of the type 1:2; 2:3; 3:4; 2:5, etc. The nonrandom nature of such a resonance structure of the satellite systems (and of the planetary system itself) forces one to look for the cause of its generation in dissipative processes. In recent times, tidal friction has been most often considered as the dissipative process [11]. A resistive medium and a nonconstant mass of the system could also be dissipative factors in the representation of A.M. Molchanov [12].

Another characteristic of the satellite systems of the Jovian planets is the closeness to the planets of the most massive satellites; their distances expressed in fractions of the radius of the sphere of influence of the planet are very small; their orbits are regular in nature, i.e. close to circles and located in the equatorial plane of the planet. Only satellites with small masses revolve at great distances from the Jovian planets and usually in very irregular orbits. The present closeness to the Jovian planets of their largest satellites is an indication that the tidal withdrawal in these systems has operated less effectively than in the Earth-Moon system and in the hypothetical systems of Mercury and Venus.

Another regularity of the satellite systems of the Jovian planets is that their chemical composition never coincides with the composition of the planets and differs within each system, if one judges from the densities of the satellites. There is evidently a general regularity of a decrease in density of the satellites with distance from the planet, similar to the regularity in the change of chemical composition of the planets with distance from the Sun. It is sharply expressed for /163
the four Galilean satellites of Jupiter, for which the masses and radii are best known. Such a regularity for the system of Saturn is detected only for the three most massive satellites - Dione, Titan, Iapetus, the data on the densities for the remaining 7 satellites are insufficiently accurate. There are no data yet for the system of Uranus; of the two satellites of Neptune, the inner satellite has the larger density.

One should indicate some other pecularities of the satellite systems of the Jovian planets. Saturn has, besides 10 satellites, a ring of solid particles revolving inside the Roche limit, which can serve as a prototype of an ideally flattened system. Jupiter has, besides its main system of 12 (sic) satellites, two groups of Trojan asteroids located in the vicinity of the triangular libration points L_4 and L_5 (cf. Figure 4.2). The influence of Jupiter on the motion in the asteroid belt (the presence of "gaps") and also on the transformation of comet orbits from long-period to short-period is well known. Thus, Jupiter controls the motion of bodies not only within its sphere of influence but also far beyond it.

Tidal evolution of the satellite systems of the Jovian planets. The role of tidal friction in the satellite systems of the four Jovian planets is clearly expressed in the synchronization of rotation of all the close and massive satellites: they turn one hemisphere toward the planets, as the Moon toward the Earth [13]. At the same time, the consequences of tidal evolution on the dimensions of the satellite orbits are scarcely perceived: they all are located within 10 or 20 times the radius of the central body, while the closest satellite usually revolves almost at the Roche limit. Meanwhile, the tidal evolution for all the satellites must have led to an expansion of the orbit, since they revolve in the same direction as the planet, and for all the satellites $\Omega > \omega$. (The Neptune-Triton system is an exception, in which Triton moves opposite to the rotation of the planet and its orbit must be contracting). With a significant role of tidal friction, the satellites could have withdrawn to distances of 10 - 20 times greater than the present distances and still revolve at distances less than half the radius of the spheres of influence of the planets. The presence of several satellites in the system of each planet permits establishing the maximum boundaries of their tidal withdrawal from the Roche limit to their present orbits. The withdrawal time of a satellite is proportional to Q of its planet and inversely proportional to the Love number k_2. By taking the duration of this allowable withdrawal equal to the duration of the existence of the solar system, i.e. $(4.5 - 4.6) \cdot 10^9$ years, and by assigning a Love number typical for solid bodies to the Terrestrial planets and values corresponding to liquid homogeneous bodies to the Jovian planets, Goldreich and Soter [13]

obtained the following values of Q (Table 10).

TABLE 10

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VALUES OF THE PARAMETER Q FOR THE DIFFERENT PLANETS:

Planet	$R \cdot 10^{-8}$, cm	$M \cdot 10^{-27}$, gm	Q	k_2
Mercury	2,4	0,32	240	0,05
Venus	6,1	4,9	17	0,25
the Earth	6,4	6,0	13*	0,30
Mars	3,4	0,65	10²*	0,10
Jupiter	71	1900	>10³***	1,5
Saturn	60	570	$>6 \cdot 10^4$	1,5
Uranus	24	87	$>5 \cdot 10^4$	0,7

* The low value of Q for the Earth is due to the large role of the oceans in dissipating tidal energy

** The estimate of Q for Mars is based here on the already obsolete data of Sharpless on the drag of Phobos (cf. [14]).

*** According to more correct estimates, k_2 for Jupiter is about 0.6 [15], thus $Q_J \geq 4 \cdot 10^4$.

It is impossible to make an estimate of Q for the Neptune-Triton system by the same method, since Triton is approaching Neptune and its initial orbit is unknown. At present, Triton revolves in retrograde motion at a distance of 13.33 times the planetary radius in an orbit inclined at 20° to the equator of Neptune. Its orbit differs very little from a circle ($e<10^{-3}$). If one assigns to Neptune the same lower limit for Q as is obtained for Uranus, i.e. $\approx 5 \cdot 10^4$ then the effect of tides on Neptune on the eccentricity of the orbit of Triton is too small to explain such a significant circularization. Only tides caused by the planet on the satellite could lead to the circularization of an initially eccentric orbit for values of $Q<500$ for Triton itself as for small solid or semi-liquid bodies.

The role of tidal friction in the origin of the commensurability of the periods of revolution in satellite systems. Simple ratios of the periods of revolution of neighboring satellites (resonance) can be preserved and maintained with withdrawal of each of the satellites from the planet due to tidal friction. Part of the orbital momentum of the inner satellite is then transmitted to the outer satellite. The commensurability is not preserved in the system without this additional transmission of momentum, because the inner satellite in the

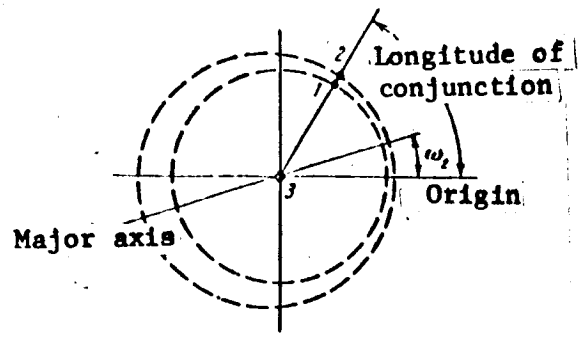

Figure 6.4. Configuration of the orbits of Titan (1) and Hyperion (2).

general case moves away relatively more rapidly than the outer, and the resonance configuration is disrupted.

The capture of one satellite into resonance with another has been studied only with the simplest models. The idea about the possibility of such capture belongs to Goldreich [11], while the mechanism for capture was studied in detail by Greenberg for the pairs of satellites of Saturn - Titan-Hyperion and Mimas-Tethys [16,17]. The Titan-Hyperion pair is convenient for consideration in that Titan is 10^3 times more massive than Hyperion. /165 Their periods of revolution are related as 3:4. The three interacting bodies - Saturn, Titan and Hyperion - differ greatly in mass: $M>>m_1>>m_2$. Consequently, the outer satellite with mass m_2 is too small to perturb m_1 or cause significant tides on Saturn. The small eccentricity of the orbit of Titan is also convenient for the model. Thus, its orbit can be considered as circular, whereas the orbit of Hyperion has a significant eccentricity.

As a result of resonance, the longitude of the conjunction of Titan and Hyperion experiences a libration about the direction to the apocenter of the orbit of Hyperion (Figure 6.4). It is convenient to describe their configuration with the help of the "resonance variable" Φ equaling:

$$\Phi = 4\lambda_2 - 3\lambda_1 - \tilde{\omega}_2, \qquad (6.8)$$

where λ is the average longitude of the satellite, $\tilde{\omega}$ is the longitude of the pericenter (the index 1 refers to Titan, the index 2 to Hyperion). At present, the value of Φ oscillates about 180° with an amplitude of 36° and a period of 640 days. The equation for the change with time of the values of Φ, e_2 and n, where $n<1$ is the ratio of the orbital periods of the satellites in the inertial reference system, can be obtained from the Lagrange equations for the changes of the orbital elements. The perturbing function characterizing the attraction of mass m_2 by mass m_1 is represented in the form of trigonometric series, from which it is sufficient to consider only the long-period terms.

$$\frac{d\Phi}{dt} = 4n - 3 - \frac{1}{2e_2}\left(\frac{m_1}{M}\right)F(n)\cos\Phi, \tag{6.9}$$

$$\frac{de_2}{dt} = -\frac{1}{2}\left(\frac{m_1}{M}\right)F(n)\sin\Phi, \tag{6.10}$$

$$\frac{dn}{dt} = 6e\left(\frac{m_1}{M}\right)nF(n)\sin\Phi - n\beta. \tag{6.11}$$

The function F (n) is a dimensionless expression of the order of unity, determined from the expansion of the perturbing function. The quantity /166 $\beta \sim$ const<0 represents the rate of change in the period because of tidal friction; it replaces the more precise expression:

$$\frac{1}{n_1^2}\frac{dn_1}{dt} = -\frac{27}{4}\left(\frac{R_p}{a_1}\right)^5\left(\frac{m_1}{M}\right)\frac{1}{Q}, \tag{6.12}$$

which can be considered constant only in the first approximation, when one can neglect the change in the semimajor axis a_1.

If there were no tidal friction in the system ($\beta=0$), then the resonance variable Φ would run through all values from 0 to 360°. For $\beta \neq 0$, the typical nonresonance system, in which Φ initially circulates over all values of angles, n is not equal to a ratio of small simple numbers and e_2 is small, evolves to the state of the type of the present system Titan-Hyperion, where Φ performs only libration, n oscillates about the value $j/(j + 1)$, where j is a small simple number, and the eccentricity e_2 increases (cf. [16] for more details). Such a state of the system is stable, when the libration of the resonance parameter Φ occurs about 180°, i.e. the conjunction of the satellites occurs close to the apocenter of m_2. Here the satellites are most widely separated and the gravitational perturbation of m_1 on m_2 is minimum. In those cases when e_2 is small, stable configurations are possible both for the conjunctions of satellites close to the apocenter of m_2 and close to its pericenter. If the satellites have comparable masses and orbital eccentricities, the description of their motions is much more complicated, but the qualitative picture for the establishment of the period commensurability remains the same as in the case under consideration. This type of resonance is called "e-resonance", since the orbital eccentricity plays the role in establishing it.

Another type of resonance motion in satellite systems is i-resonance, encountered only for close satellites, whose orbits precess under the effect of the equatorial bulge of the planet. The only known example

of this kind is the pair of satellites of Saturn - Mimas-Tethys, whose periods are related as 2:1. The resonance variable here:

$$\psi = 4\lambda_T - 2\lambda_M - \Omega_T - \Omega_M, \qquad (6.13)$$

where Ω is the longitude of the angle of the corresponding satellite in the equatorial plane of Saturn. The conjunction of the satellites oscillates about the average between the nodes of their orbits [17]. There is no doubt about the role of tidal friction in establishing the described resonances in the systems of satellites of the Jovian planets. It is not clear how general such a picture is. Perhaps, in the case of the motion of massive satellites around terrestrial planets, where the tidal evolution occurs more rapidly, resonances should be considered only as a particular case.

The effect of tidal friction on the ordering of the orbital inclinations of satellites with the system of satellites of Uranus as an example. The system of Uranus is significant with its own regularity: the five satellites are arranged in almost circular orbits practically precisely in the equatorial plane of the planet, whereas the axis of Uranus is inclined to the axis of the ecliptic by 98°. /167

One can calculate that the precession of the axis of Uranus under the influence of the solar effect on the equatorial bulge of Uranus is very gradual: one revolution of the axis is accomplished in a period of more than 10^{10} years. With this slow precession of the axis of the planet, the orbits of the satellites preserve the constant orientation to its equator [18]. Thus, if the orbits of the satellites were inclined at some time to the equator of Uranus, then tidal friction can be considered the only conceivable cause forcing them to "lie" on the equator [19]. We do not know the properties of the material of Uranus, which determine the rate of tidal evolution of the satellite orbits. However, it can be assumed that over the time of existence of the solar system the change in the satellite orbits could not be greater than the withdrawal of each of the satellites from the orbit of the preceding, and the withdrawal of the closest of the five satellites - Miranda is no more than the withdrawal from the Roche limit (2.5 - 3) R to its present distance of about 5R, where R is the radius of Uranus. Thus, we can specify the limits of the change in a_1, where a_1 is the semimajor axis of the satellite orbit. By making use of the fact that

the mutual tidal effect of the satellites is small, we consider the change in the angle of inclination ε_i of the orbit of each of the five satellites to the equator of Uranus for the case of two bodies: the given satellite and the planet.

It is more convenient to take the distance of the satellite from the planet (or the semimajor axis of its orbit) as the independent variable in place of the time. In this case, the change in ε can be evaluated by applying the formula of Mac Donald (3.7), which was derived by him for the case when the tidal torque is proportional to the tidal phase lag:

$$\frac{d\eta}{da} = -\frac{B(q)\sin\varepsilon}{2a(\cos\varepsilon-\alpha)F(q)}. \qquad (6.14)$$

A relation for $d\eta/da$, analogous to (6.14), was derived by M.L. Lidov [20] for the case when the tidal torque is proportional to the difference $(\Omega - \omega)$:

$$\frac{d\eta}{da} = -\frac{1}{4a}\frac{\sin\varepsilon}{\cos\varepsilon-\alpha}. \qquad (6.15)$$

Relation (6.14) becomes (6.15) for small q^2.

The main angular momentum in the system of Uranus is included in the rapidly rotating planet itself, i.e. the angle $\varepsilon - \eta$ is very small. With the tidal withdrawal of the satellites, the axis of the planet changes its position in space very insignificantly; the angle $\varepsilon - \eta$ changes only by several seconds of arc. This means that during the course of the tidal evolution, the equatorial plane of Uranus can be assumed to be a fixed plane, i.e. one can assume $\varepsilon - \eta \tilde{\approx} 0$. The angular velocity of the axial rotation of Uranus is also very small. Thus, we set: /168

$$\varepsilon \approx \eta, \quad \Omega \approx \text{const}. \qquad (6.16)$$

By specifying the arbitrary "initial" values of the angle ε, one can obtain its change during the course of maximum withdrawal of the satellites. The values of the present distances of the satellites of Uranus are presented below:

No. of satellite	5	1	2	3	4
a_i/R_u	5,11	7,52	10,18	17,21	23,01

Under the assumption that each satellite could have withdrawn only from the orbit of the preceding to its own orbit, all the distances

could have increased by no more than 2 times.

In the case of small "initial" values of $\varepsilon(\varepsilon \lesssim 20)$ and for $\alpha << 1$, the simpler formula (6.15), with consideration of (6.16) gives:

$$\varepsilon \cong \varepsilon_0 (a_0/a)^{1/4}. \qquad (6.17)$$

Hence, it follows that for an increase of (a) by 2 times, the angle ε decreases by approximately 20%. If, for example, the initial inclinations were 20°, then the present inclinations would be about 16°, and not 0°, as they actually are.

For $\varepsilon_0 > 20°$, the change in the angles of inclination should be calculated from Formula (6.14). For all the satellites, the decrease in the "initial angles of inclination taken in the interval $20° < \varepsilon < \arccos \alpha$, as a rule, do not exceed 10°.

For $\cos \varepsilon - \alpha < 0$, i.e. for $\varepsilon > \varepsilon_{cr} = \arccos \alpha$, the derivative $d\eta/da$ is positive. However, for this $da/dt < 0$[9], i.e. approach of the satellite toward the planet due to tidal friction occurs, and not withdrawal. Consequently, the inclination of the orbit decreases with time even in this case. For each circular orbit characterized by the quantity $\alpha = \omega/\Omega$, there exists only one value of ε_{cr}. Values of ε_{cr} are presented below for the radii of the circular orbits equaling the Roche limit and the radii of the orbits of the five satellites of Uranus:

a_i/R_u	3	5	7	10	17	23
$\alpha = \omega/\Omega$	0,741	0,345	0,208	0,122	0,055	0,035
ε_{cr}, deg	42,1	69,8	78,0	83,0	86,8	88,0

The largest change in ε for a given change in (a) could occur close to $\varepsilon = \varepsilon_{cr}$. It is extremely unlikely that the initial orbits of the satellites were actually located in just the planes with the critical inclinations. Nonetheless, it is interesting to evaluate the change /169 in the angle ε for such specially selected values of ε_0, so that an estimate is obtained of its necessarily maximum change. Figure 6.5 presents examples of the calculations of the dependence $\eta(a)$ for the intervals of change in the semimajor axis of the satellite orbits of $3R \leq a_5 \leq 5R$ (curve 1) and $10R \leq a_3 \leq 17R$ (curve 2). For $\varepsilon = \varepsilon_{cr}$,

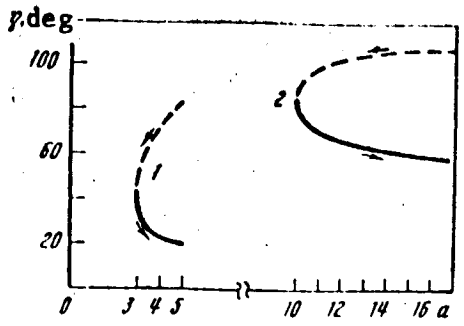

Figure 6.5. Examples of the amximum changes in the angles of inclination of the orbits of the satellites of Uranus to the plane of its equator; the arrows indicate the direction of change in the angles.

the curve η(a) has a vertical tangent and two branches: along one of them the angle η increases with (a), along the other it decreases. The angle ε also behaves thusly in our case. But the absolute changes in ε are all insufficient here to give ε = 0, i.e. the coincidence of the orbital planes of the satellites and the equator of the planet. Calculations for satellites with other possible intervals of changes in (a) lead to an analogous picture, where the absolute value of the changes in η are smaller than shown in Figure 6.5.

There is a particular interest in the "initial" arrangement of the planes of the satellite orbits in the ecliptic, i.e. $\varepsilon_0 = 98°$. Since such satellites would be retrograde with respect to the rotation of Uranus, tidal friction would lead to their approach toward the planet, i.e. da/dt < 0. In this case, the value of dη/da in Equations (6.14) and (6.15) would be positive, since $(\cos 98° - \alpha) \ll 0$, and we would have at first a contraction of the orbital radii with a decrease in the angle ε, but only until $\varepsilon = \varepsilon_{cr}$ (an idea was given above about the numerical values of α and ε_{cr} in the present system of Uranus). The approach toward the planet must then change to separation from it and a decrease in ε. Then we would have withdrawal, limited by the limits of the present orbits and not leading to ε = 0°.

One can conclude that the present arrangement of the satellites of Uranus in the plane of its equator is not a result of the tidal evolution of initially inclined orbits. The regular structure of the system of Uranus must evidently have arisen in the process of its accumulation.

6.3. Origin of the Satellites of the Jovian Planets

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The problem of the origin of the Jovian planets and the role of gas in their formation*. The main problem of the origin of Jupiter and Saturn is the question of how these planets absorbed the lightest gases - hydrogen and helium, which became their predominating components. Theoretically, there are three possibilities: 1) the condensation of all the gases of the preplanetary cloud onto solid particles and then the accumulation of these particles into the planets; 2) the formation of gas-dust clumps in the preplanetary cloud as a result of its gravitational instability; 3) accretion of gases on the nuclei of the Jovian planets, which were being formed by the accumulation of solid particles composed of nonvolatile components.

The first version requires unacceptably low temperatures. The condensation of hydrogen into the solid phase at a density characteristic for the zone of Jupiter could have occurred at temperatures no greater than 4 - 5°K and for the condensation of helium would have required a temperature much less than 1°K. It is obvious that the condition for the condensation of helium into the solid phase is generally inaccessible in the galaxy.

Thus, the possibility of the condensation of hydrogen (without helium) was studied under the conditions of the preplanetary cloud. It was assumed that temperatures of 4 - 5°K could have been achieved with screening of the solar radiation. Models of Jupiter from hydrogen and heavier elements, excepting helium, were constructed from just such a hypothesis [22]. Öpik [23], considering the main mass of Jupiter as consisting of hydrogen and the atmosphere consisting of helium, assumed that there could have been formed in the preplanetary cloud snowflakes of H_2, which precipitated with the other condensates through the gaseous helium during the course of the entire process of growth of Jupiter (of course, such "snowflakes" must have evaporated close to the planet, which was heated due to the gravitational energy). According to Öpik, Jupiter captured the helium atmosphere at the end of its growth. However, the atmosphere of Jupiter has a composition

* The contents of this paragraph are discussed in [21].

similar to the composition of the main mass of the planet, i.e. predominantly hydrogen. Moreover, the assumption about the condensation of hydrogen encounters difficulties. A.I. Lebedinskiy [24] showed in 1960 that the energy of motion of large solid bodies in a gas-dust cloud, by agitating the motion of small dust particles and increasing the frequency of mutual collisions, must heat it to temperatures from 5 to 30°K. In 1962, V.S. Safronov [25] evaluated the temperature of the preplanetary cloud with consideration of heating by solar radiation scattered in the gaseous component of the cloud, assuming the solar radiation to be equivalent to that at present. It was found that for the present luminosity of the Sun, the temperature of the dust layer in the zone of Jupiter must have been no less than 35°K and in the /171
zone of Saturn 18°K. The temperature of the gas component was even higher. Condensation of hydrogen is excluded at such temperatures. Thus, the first version for the formation of the Jovian planets has to be rejected.

The second version - gravitational instability in the gas component of the preplanetary cloud (H_2 + He) - requires too great a mass of the preplanetary cloud.

For the collapse into self-gravitational clumps, the gas density ρ_0 close to the central plane of the cloud must have reached a critical value, which exceeds by 10 times the "smeared" density of the Sun: $\rho^* = M_\odot/4/3\pi R^3$, where R is the distance of Jupiter from the Sun. Since the initial composition of the cloud is close to that of the Sun, the dust component was only 1 - 2% by mass, and it cannot be taken into account in evaluating the critical density in the gas. At a temperature of 35°K in the zone of Jupiter and 18°K in the zone of Saturn, the total mass of material in the zones must have been no less than $5 \cdot 10^{32}$ gm $\approx 0{,}25 M_\odot$. This is equivalent to 250 times the mass of Jupiter. Even if it is assumed that the gravitational instability developed with the help of ring-shaped perturbations causing the collapse of part of the cloud into a ring, which then collapsed into clumps (the critical density for this $\rho_{cr} \geq 2.1\rho^*$ [26]), then the mass of the gas in the zone of Jupiter and Saturn must have been at least $2.4 \cdot 10^{32}$ gm $\approx 0{,}12 M_\odot$, or 120 times the mass of Jupiter. The perturbations of Jupiter and Saturn could have ejected from the solar system only bodies with a

total mass no more than 10 times the mass of Jupiter [27]. It is difficult to understand how one could get rid of the remaining, more than 100 times the mass of Jupiter.

One can conclude that the formation of clumps of solar composition in the zones of Jupiter and Saturn did not occur, i.e. this version also does not fit. It should be added that for the generation of the other planets of the solar system, which do not have "solar," but another composition, the mechanism of gravitational instability in the gas is all the more inapplicable.

The third version - the accretion of gas by the growing planet - is evidently the only acceptable version. The growth process in this case consists of two stages. Only the accumulation of solid particles and bodies occurs at the first stage in the preplanetary cloud of solar composition, while the gas, since it is present, plays the role of only a resisting medium. The second stage begins when the solid nucleus of the planet reaches some mass M_0: the gas is directed toward the nucleus and combines with it, forming an increasingly thick shell. The equation for axially symmetric accretion was already derived in 1939 by Hoyle and Lyttleton, as applied to the capture of interstellar gas by stars. In 1946, Hoyle [28] applied it to the process of the condensation of the planets, by substituting the velocity of the planets with respect to the gas for the average thermal velocity of the gas atoms. In 1952, Bondi [29] derived the equation /172 for spherically symmetric accretion, which practically coincided with that applied by Hoyle. The formula for the increase in mass due to accretion is very simple:

$$\frac{dM}{dt} = \frac{4\pi\lambda (GM)^2 \rho_\infty}{c^3}. \qquad (6.18)$$

Here ρ_∞ and c are the density of the gaseous medium and the velocity of sound far from the gravitational body, while λ is a numerical parameter, which depends on the exponent γ:

$$\frac{p}{p_\infty} = \left(\frac{\rho}{\rho_\infty}\right)^\gamma, \qquad (6.19)$$

where ρ_∞ is the gas pressure far from the body. We have $1.12 \leq \lambda \leq 0.25$ for the interval $1 \leq \gamma \leq 5/3$.

The accretion of gas can formally begin for that mass of the nucleus, when its gravitational cross section πr_c^2 becomes greater than the geometric cross section πr^2. This mass is $10^{24} - 10^{25}$ gm for thermal velocities of the atoms of about 1 km/sec. However, as was shown by B.S. Safronov [26], the mass for the beginning of effective accretion must be $10^3 - 10^4$ times greater, i.e. it must be at least 1 or 2 times the mass of the Earth or the order of 1% of the mass of Jupiter and Saturn. Dole [30] later estimated this critical mass as 3 - 6 times the mass of the Earth. This estimate followed from a comparison of the rate of increase of the gaseous shell with its leakage rate due to thermal dissipation. Only for a value of $M_0 \approx M_\odot$ is a thick atmosphere formed with an infinitely increasing mass and increasing density, which collects all the surrounding gas, instead of a steady-state atmosphere of relatively small mass. Hoyle had already noted that accretion is a rapidly accelerating process, which is accompanied by a great release of energy. Formula (6.18) was used by V.S. Safronov [31] to evaluate the initial temperature of Jupiter and Saturn, which was very high (up to 17,000°K for Jupiter and up to 3600°K for Saturn).

All these calculations have indicated the possibility in principle of forming Jupiter and Saturn in two stages with rapid accretion of gas in the final stage. However, there is no complete theory of the growth of the Joivan planets. It is difficult to construct because the supply zones of these planets intersected; some part of the material was ejected from the solar system. The hydrodynamics of accretion are still insufficiently studied. Nonetheless, to construct a model of the formation of the satellites of the Joivan planets, we will begin with the simplest theory of the accretion of the planets themselves.

The formation of the satellites of the hydrogen-helium planets. The formation of the satellite swarms around the Jovian planets Jupiter and Saturn must have occurred both at the stage of growth of the solid nucleus of the planet (the stage like that of the growth of the terrestrial planets) and at the stage of gas accretion. During the first /173
stage, both the planet and the swarm were supplied by the same source - solid bodies and particles consisting of condensed material. At the second stage, the planet rapidly captured gas in addition to solid

bodies, while the supply of the swarm continued only due to the solid component. In evaluating the total mass flowing through the swarm, we must take into account both stages of its growth. But that portion of the swarm, which was collected at the first stage, could not be maintained in the form of satellites. The accretion of gas must have increased the mass of the planet by tens or hundreds of times as compared to the mass of the solid nucleus. The dimensions of the satellite orbits must have then contracted in accordance with the Jeans invariant $m_0 l_0 = ml$, where m_0 is the initial mass of the planet, l_0 is the initial radius of the satellite orbit, and m and l are their current values. All satellite orbits for nuclei with masses the order of that of the Earth must have been located within several tens of the radius of the nucleus of the planet. The contraction of the dimensions of the orbits by 10^2 times leads inevitably to the absorption by the planet of all the material of the swarm acquired during the first stage of growth.

Consequently, in evaluating the mass of the present satellite systems, we can neglect the first stage and consider only the second stage - the accretion of gas, when the supply of the planet and swarm came from different sources. This is the basic difference in the formation of the satellites of the hydrogen-helium planets and of the terrestrial planets. We have for the mass of the swarm μ:

$$\frac{d\mu}{dt} = \frac{d\mu_1}{dt} + \frac{d\mu_2}{dt} + \frac{d\mu_3}{dt}, \qquad (6.20)$$

where μ_1 is the contribution of the "free-free" collisions, while μ_2 is the contribution of "free-bound" collisions (cf. Chapter 4). The term with μ_3 must be added here in view of the obvious enrichment of the swarm due to the sticking of dust particles in the gas flux flowing toward the growing planet. Its evaluation requires a more detailed knowledge of the hydrodynamics of the process of accretion, than we have at present. If the dust density ρ_d is assumed proportional to the gas density ρ_g at all stages of the accretion process, then the maximum increase in the mass of the swarm will be the same, as in the case of the terrestrial planets, for the mass of the planet of about $(2/5 - 1/2)M_{pres}$. However, only the small particles are actually dragged away by the gas and contribute to μ_3, while the large bodies move independently of the gas flux. The proportionality $\rho_d \sim \rho_g$ is not observed because of the different rate in scooping up of the gas and solid material.

Let us dwell on the estimate of the planet-centered momentum of the satellite swarms of the hydrogen-helium planets. The participation of the gas in their formation gives the basis for using the hydrodynamic estimate of the momentum of the swarm (+ $1/4I\omega_c$, version I) rather than the estimate from the hypothesis of "solid-body" rotation (+ $I\omega_c$, version II), which we used for the swarms of the Terrestrial planets (cf. Chapter 4.5). In the case of the Terrestrial planets, we obtained in /174
section 1 of this chapter very tentatively that the equivalent distance of a direct satellite from the planet in fractions of its radius is proportional to the distance from the Sun in astronomical units ($l \sim \mathfrak{R}$).

By taking $l_\odot = 20R_\odot$ as the reference of the initial distance of the satellite, we obtain the following initial distances l for the satellites of Jupiter and Saturn in the two versions for evaluating the specific momentum of the swarm:

l_1	$740R_J$	$1100R_S$
l (Version I)	$6R_J$	$11R_S$
l (Version II)	$160R_J$	$180R_S$

The present massive satellites of Jupiter are arranged at distances from $6R_J$ to $27R_J$, while the "center of gravity" corresponds to $\tilde{\sim}15R_J$. The massive satellites of Saturn revolve at distances from 3 to $60R_S$ with a "center of gravity" at $\tilde{\sim}20R_S$. Both of the formally obtained "centers of gravity" of the satellite systems fall within the interval between the values for the distance of the "equivalent" satellite for the extreme cases of the dependence of k on ω_s. As was previously noted, tidal friction changed little the initial dimensions of the satellite orbits of the Jovian planets. Thus, the conversion to their present dimensions is valid here.

In summing up, it can be noted that a theory for the formation of the satellite systems in all their variety has yet been produced. This chapter can be considered only as a first attempt to construct such a theory, by using the relations derived in studying the formation of the Moon with the accumulation of the Earth. With the growth of knowledge about processes occurring at the periphery of the solar system in its early stage, one can look forward to further development of this model for the formation of the satellites.

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CONCLUSION

The investigation of the solar system with the help of space technology successfully moves forward. /176 Tremendous progress is noted in experimental and theoretical investigations of the properties of material under conditions close to those in space; in the technique of radioactive dating of rock samples; in the application of contemporary computer technology for studying the behavior of complex dynamic and physical systems. Of course, knowledge about the Earth has become more abundant. All of this permits one to assume that in the near future a correct representation will be worked out for the origin and development of the solar system and the Earth. The cosmogony of the Moon and the satellites of the planets will always remain an important division of planetary cosmogony. The choice of theoretical models for the formation of the Moon is still rather wide, since a unique interpretation of the data on the Moon is a very complex problem. We have presented arguments in favor of the model for the formation of the Moon in an orbit about the Earth during the process of growth of the Earth. The satellites in this model are the final product of the "assembly" of particles and bodies, which formed satellite swarms about the growing planets. Little studied problems include the chemical composition and regularity in its change among the satellites; an evaluation of the planet-centered angular momentum of the satellite swarms, investigation of the motions in the gas during the process of growth of the planets along with the satellite swarms, the dynamics of bodies at the periphery of the solar system, where the mass of ejected bodies exceeds the mass of material accumulated into the planets, the growth of Uranus and Neptune and the pecularities in the formation of their satellites.

We hope that these questions can be solved in the future, without changing the essence of the models discussed in this book.

Appendix to Chapter 2.3

FORMULATION OF THE PROBLEM ON THE TIDAL EVOLUTION OF A CIRCULAR LUNAR ORBIT INCLINED TO THE EQUATOR OF THE EARTH, ACCORDING TO P. GOLDREICH

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The precession equations. Let us first consider the motion on the short and intermediate time scales. For simplicity, one can assume that the Earth is an oblate ellipsoid of rotation with principal moments of inertia:

$$C = I\,[1 + (2k, R^5/9GI)\,\Omega^2], \tag{1}$$

$$A = I\,[1 - (k, R^5/9GI)\,\Omega^2], \tag{2}$$

where R is the equatorial radius, I is the moment of inertia of the equivalent sphere, k is the so-called Love secular number equaling $k_s = 0.947$ according to [9].

The free nutation of the axis of the Earth can be neglected in this problem, by assuming that the direction of its axis of rotation coincides with the direction of the angular momentum. Forced nutation under the effect of the Moon (its period is now about 18 years) remains in the equations.

The potential of the perturbing forces acting on the lunar orbit due to the oblateness of the figure of the Earth, can be described in the form [30]:

$$R_1 = \frac{\mu}{r}\left[\frac{A_2}{r^2}P_2(\sin\theta) + \frac{A_3}{r^3}P_3(\sin\theta) + \ldots\right], \tag{3}$$

where $\mu = G(M + m)$, r is the Earth-Moon distance, θ is the latitude, and the coefficients A_n are determined by the deviation of the figure of the Earth from a sphere; because of axial symmetry $A_3 = 0$. With an accuracy to $A_2 = -2/3JR^2$, where $J = 3/2(C - A)/MR^2$:

$$R_1 = -\frac{\mu}{r}\left[+\frac{2}{3}\frac{JR^2}{r^2}P_2(\sin\theta)\right]. \tag{4}$$

The perturbing potential from the Sun acting on the Moon can be written in the form [31]:

$$R_2 = \frac{\mu}{r}\left(\frac{M_\odot}{M+m}\right)\left[\frac{r^2}{r_\odot^3}P_2(\cos S) + \left(\frac{M-m}{M+m}\right)\frac{r^4}{r_\odot^4}P_3(\cos S) + \ldots\right]. \tag{5}$$

Here $r_\odot$ is the Earth-Sun distance; S is the angle between the Earth-Moon and Earth-Sun directions; $M_\odot$, M and m are the masses of the Sun, Earth and Moon respectively. As in the case of the potential R_1, one can be limited to terms with P_2 only. Since the motions of the Moon and the Sun are circular by assumption, then $r = a$, $r_\odot = a_\odot$. /178

The expressions for R_1 and R_2 should be averaged on the short time scale, in order to isolate the "secular" parts in them. It is necessary for this to express the angles θ and S in terms of the Keplerian elements of the lunar and solar orbits with respect to the Earth. Figure 2.9 shows the coordinate systems with the center at the Earth, which are fixed to the planes of motion of the Moon (X_1X_2) and the Sun (Y_1Y_2). We denote by i the angle between these planes, and by φ' the angular coordinate on the orbit of the Moon measured from the node of the orbit (X_1X_2) on the ecliptic (Y_1Y_2). The unit vectors a, b, c denote the normals to the planes of the equator, the lunar orbit and the ecliptic respectively. We denote by ε the inclination of the lunar orbit to the plane of the terrestrial equator (the angle between a and b in Figure 2.9, a) and by γ the angle between the equator and the ecliptic (the angle between a and c in Figure 2.9, b). Let φ and u be the angular coordinates of the Moon and Sun in the planes of their orbits, measured respectively from the axes X_1 and Y_1, as is shown in the figures. Then one can express the angles θ and S in terms of the orbital elements and obtain:

$$\sin\theta = \sin\varepsilon \sin\varphi, \tag{6}$$

$$\cos S = \cos^2\frac{i}{2}\cos(\varphi' - u) + \sin^2\frac{i}{2}\cos(\varphi' + u). \tag{7}$$

By substituting (6) into (4), we have:

$$R_1 = \frac{2}{3}\frac{\mu}{a^3}JR^2\left(\frac{1}{2} - \frac{3}{4}\sin^2\varepsilon + \frac{3}{4}\sin^2\varepsilon\cos 2\varphi\right).$$

In averaging over the period of revolution of the Moon, the term with 2φ vanishes, and we obtain the secular part of R_1:

$$R_1 = \frac{2}{3}\frac{\mu}{a^3}JR^2\left(\frac{1}{2} - \frac{3}{4}\sin^2\varepsilon\right). \tag{8}$$

One can express the secular part of (5) in an analogous manner with the help of (7) in the form:

$$R_2 = \mu\left(\frac{M_\odot}{M+m}\right)\frac{a^2}{a_\odot^3}\left(\frac{1}{4} - \frac{3}{8}\sin^2 i\right). \tag{9}$$

One can write by analogy to (8) the secular part of the perturbing potential "seen" by the Sun from the oblateness of the Earth:

$$\bar{R}_3 = \frac{2GMJR^2}{3a_\odot^3}\left(\frac{1}{2} - \frac{3}{4}\sin^2\gamma\right). \tag{10}$$

The averaging of the motions of the Moon and Sun over the short time scale implies physically the replacement of their masses by rotating toruses equivalent in mass and located in the planes of the corresponding orbits. One can find the mutual torque between the Earth, Moon and Sun, by differentiating $\bar{R}_1$, $\bar{R}_2$, $\bar{R}_3$ with respect to ε, i and γ and multiplying by the corresponding masses (m or $M_\odot$). In vector form: /179

$$\mathbf{L}_{\mathbb{C}\oplus} = -\frac{\mu m}{a^3} JR^2 \sin\varepsilon\cos\varepsilon \frac{\mathbf{a}\times\mathbf{b}}{|\mathbf{a}\times\mathbf{b}|},$$

or, by taking into account that $\sin\varepsilon = |\mathbf{aXb}|$,

$$\mathbf{L}_{\mathbb{C}\oplus} = -\frac{\mu m}{a^3} JR^2 (\mathbf{a}\cdot\mathbf{b})(\mathbf{a}\times\mathbf{b}). \tag{11}$$

By analogy:

$$\mathbf{L}_{\odot\oplus} = -\frac{GMM_\odot}{a_\odot^3} JR^3 (\mathbf{a}\cdot\mathbf{c})(\mathbf{a}\times\mathbf{c}), \tag{12}$$

$$\mathbf{L}_{\mathbb{C}\odot} = +\frac{3}{4}\mu\left(\frac{M_\odot m}{M+m}\right)\frac{a^2}{a_\odot^3}(\mathbf{b}\cdot\mathbf{c})(\mathbf{b}\times\mathbf{c}). \tag{13}$$

The equations for the torques should be supplemented by the obvious relations:

$$\begin{aligned}
\mathbf{L}_{\oplus\mathbb{C}} &= -\mathbf{L}_{\mathbb{C}\oplus} = L(\mathbf{a}\times\mathbf{b})(\mathbf{a}\cdot\mathbf{b}),\\
\mathbf{L}_{\odot\odot} &= -\mathbf{L}_{\odot\oplus} = -K_1(\mathbf{a}\times\mathbf{c})(\mathbf{a}\cdot\mathbf{c}),\\
\mathbf{L}_{\mathbb{C}\odot} &= -\mathbf{L}_{\odot\mathbb{C}} = K_2(\mathbf{b}\times\mathbf{c})(\mathbf{b}\cdot\mathbf{c}),
\end{aligned} \tag{14}$$

where the scaler quantities L, K_1, K_2 are respectively:

$$L = \frac{\mu m}{a^3} JR^2, \quad K_1 = \frac{\mu M_\odot}{a_\odot^3} JR^2, \quad K_2 = \frac{3}{4}\mu\left(\frac{M_\odot m}{M+m}\right)\frac{h^2}{a_\odot^3}. \tag{15}$$

If one now introduces the scalar symbols H and h for the magnitudes of the angular momentum vectors for the axial rotation of the Earth and the orbital motion of the Moon, then one can write the equations for their change in the form:

$$\frac{d(H\mathbf{a})}{dt} = \mathbf{L}_{\oplus\mathbb{C}} + \mathbf{L}_{\oplus\odot}, \quad \frac{d(h\mathbf{b})}{dt} = \mathbf{L}_{\mathbb{C}\odot} + \mathbf{L}_{\mathbb{C}\odot}, \tag{16}$$

and with the substitution of (14) in the form:

$$\begin{aligned}
\frac{d(H\mathbf{a})}{dt} &= L(\mathbf{a}\times\mathbf{b})(\mathbf{a}\cdot\mathbf{b}) - K_1(\mathbf{c}\times\mathbf{a})(\mathbf{a}\cdot\mathbf{c}),\\
\frac{d(h\mathbf{b})}{dt} &= -L(\mathbf{a}\times\mathbf{b})(\mathbf{a}\cdot\mathbf{b}) + K_2(\mathbf{b}\times\mathbf{c})(\mathbf{b}\cdot\mathbf{c}).
\end{aligned} \tag{17}$$

Hence, one can obtain the expressions for the scalar invariants in the short and intermediate time scales, i.e. without consideration of the effect of tidal friction.

For this, we take the scalar product of Equations (16) with a and/180 b respectively. The right sides of the equations vanish because a and b respectively appear in the mixed vector products. We have from the first equation of (16):

$$\mathbf{a}\frac{d(H\mathbf{a})}{dt} = 0 = \mathbf{a}\cdot\mathbf{a}\frac{dH}{dt} + H\mathbf{a}\cdot\frac{d\mathbf{a}}{dt} = \frac{dH}{dt} = 0. \tag{18}$$

Consequently, H = const. From the second equation of (16), we obtain by analogy:

$$\frac{dh}{dt} = 0, \tag{19}$$

i.e. h = const.

If one now takes the scalar product of both equations (16) with c and uses (18) and (19), then:

$$H\frac{d(\mathbf{a}\cdot\mathbf{c})}{dt} = L(\mathbf{a}\cdot\mathbf{b})(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}, \tag{20}$$

$$h\frac{d(\mathbf{b}\cdot\mathbf{c})}{dt} = -L(\mathbf{a}\cdot\mathbf{b})(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}, \tag{21}$$

Hence:

$$\frac{d\lambda}{dt} = \frac{d}{dt}[H(\mathbf{a}\cdot\mathbf{c}) + h(\mathbf{b}\cdot\mathbf{c})] = 0, \tag{22}$$

i.e. λ = const.

Thus, on the short and intermediate time scales, i.e. in the absence of tidal friction, the scalar magnitudes of the axial momentum of the Earth H, the orbital momentum of the Moon h and the projection of the total angular momentum of the Earth-Moon system onto the axis of the ecliptic λ are conserved. A fourth and last scalar invariant can be obtained, if one forms the combination hHd(a·b)/dt:

$$\frac{d(\mathbf{a}\cdot\mathbf{b})}{dt} = \left[\frac{K_2}{h}(\mathbf{b}\cdot\mathbf{c}) - \frac{K_1}{H}(\mathbf{a}\cdot\mathbf{c})\right](\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}. \tag{23}$$

If one now multiplies Equation (20) by 2 (a·c)/H, Equation (21) by 2 (b·c)/h and Equation (22) by 2 (a·b), and then adds them, then this gives:

$$\frac{d\chi}{dt} = \frac{d}{dt}[K_1(\mathbf{a}\cdot\mathbf{c})^2 + K_2(\mathbf{b}\cdot\mathbf{c})^2 + L(\mathbf{a}\cdot\mathbf{b})^2] = 0. \tag{24}$$

According to the form, this is the sum of potential energies. The integral of the energy χ = const is a natural consequence of the conservation on the short and intermediate time scales of the scalar momenta H and h, i.e. conservation separately of the kinetic energy of rotation and revolution. All four invariants H, h, λ and χ change on the long time scale.

Let us turn to the scalar form in describing the equations of motion. We denote by w the scalar triple product (a X b) · c. Then one can write: /181

$$w^2 = |(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}|^2. \tag{25}$$

By setting x = (a · c), y = (b·c), z = (a·b), w = (a X b) c, we obtain:

$$w^2 = 1 - x^2 - y^2 - z^2 + 2xyz, \tag{26}$$

and also the remaining equations in scalar form:

$$\lambda = Hx + hy, \tag{27}$$

$$\chi = K_1 x^2 + K_2 y^2 + L z^2, \tag{28}$$

$$\frac{dx}{dt} = \frac{L}{H} zw, \tag{29}$$

$$\frac{dy}{dt} = -\frac{L}{h} zw, \tag{30}$$

$$\frac{dz}{dt} = \left(\frac{K_2}{h} y - \frac{K_1}{H} x\right) w, \tag{31}$$

$$w^2 = 1 - x^2 - y^2 - z^2 + 2xyz. \tag{32}$$

In order to avoid the procedures for determining the sign of w as the square root with the solution of (32), one can derive another equation for w by using the remaining relations:

$$\frac{dw}{dt} = \frac{L}{H} z(yz - x) - \frac{L}{h} z(xz - y) + \left(\frac{K_2}{h} y - \frac{K_1}{H} x\right)(xy - z). \tag{33}$$

Four equations are sufficient to find all the unknowns, for example, (27), (29), (31), (33). The variability of the quantities x,y,z,w has a periodic nature. It is not difficult to show that w is positive for half of the precession period, negative the other half and passes through zero twice during the precession period.

Equations of tidal friction. Let us now turn to the long time scale and introduce the tidal torques. Equations (16) take the form:

$$\frac{d(H\mathbf{a})}{dt} = \mathbf{L}_{\oplus\mathfrak{D}} + \mathbf{L}_{\oplus\odot} + \mathbf{T}_{\oplus}, \tag{34}$$

$$\frac{d(h\mathbf{b})}{dt} = \mathbf{L}_{\mathfrak{D}\oplus} + \mathbf{L}_{\mathfrak{D}\odot} + \mathbf{T}_{\mathfrak{D}}, \tag{35}$$

where $T_\odot$ is the resultant tidal torque acting on the Earth; T_ζ is the torque acting on the Moon. By taking the scalar product of the first of these equations with a, and the second with b, we obtain (cf. (180):

$$\frac{dH}{dt} = T_\odot \cdot a, \tag{36}$$

$$\frac{dh}{dt} = T_\zeta \cdot b. \tag{37}$$

By taking the scalar product of both equations with c, we have: /182

$$H\frac{dx}{dt} = T_\odot \cdot c - xT_\odot \cdot a, \tag{38}$$

$$h\frac{dy}{dt} = T_\zeta \cdot c - yT_\zeta \cdot b. \tag{39}$$

By taking the scalar products of Equations (34) and (35) with b and a respectively and adding, we have:

$$\frac{dz}{dt} = \frac{T_\odot \cdot b}{H} + \frac{T_\zeta \cdot a}{h} - z\left(\frac{T_\odot \cdot a}{H} + \frac{T_\zeta \cdot b}{h}\right). \tag{40}$$

It follows from Equations (36) - (39):

$$\frac{d\lambda}{dt} = (T_\odot + T_\zeta)\cdot c. \tag{41}$$

An equation for da/dt can be obtained from (37) for dh/dt. Since $h = m(\mu a)^{1/2}$, then:

$$\frac{da}{dt} = 2aT_\zeta b/h. \tag{42}$$

By using (1), (4) and (17), one can find:

$$\frac{dK_1}{dt} = \frac{2K_1}{H}\frac{dH}{dt} = \frac{2K_1}{H}T_\odot \cdot a, \quad \frac{dK_2}{dt} = \frac{2K_2}{a}\frac{da}{dt} = \frac{4K_2T_\zeta b}{h},$$
$$\frac{dL}{dt} = \frac{2L}{H}\frac{dH}{dt} - \frac{3L}{a}\frac{da}{dt} = \frac{2L}{H}T_\odot \cdot a - \frac{6LT_\zeta \cdot b}{h}. \tag{43}$$

Hence:

$$\frac{d\chi}{dt} = \frac{2K_1}{H}xT_\odot \cdot c + \frac{2K_2}{h}(T_\zeta \cdot c + yT_\zeta \cdot b) + \\ + 2Lz\left(\frac{T_\odot \cdot b}{H} + \frac{T_\zeta \cdot a - \frac{1}{2}zT_\zeta \cdot b}{h}\right). \tag{44}$$

If the torques $T_\odot$ and T_ζ are resolved along the coordinate axes $X_1X_2X_3$ and $Y_1Y_2Y_3$ (cf. Figure 2.9, a and b), then one can present the equations of motion for the long time scale in scalar form. Let the unit vectors along the coordinate axes be e_1, e_2 and e_3 respectively. They can be expressed in terms of a, b and c:

$$e_1 = \frac{a \times b}{\sin\varepsilon}, \quad e_2 = \frac{a - \cos\varepsilon b}{\sin\varepsilon}, \quad e_3 = b, \tag{45}$$

$$f_1 = \frac{a \times c}{\sin\gamma}, \quad f_2 = \frac{a - \cos\gamma c}{\sin\gamma}, \quad f_3 = c. \tag{46}$$

Each of the two torque vectors $T_\odot$ or T_ζ can be expressed in general form as:

$$\mathbf{T} = T_1\mathbf{c}_1 + T_2\mathbf{c}_2 + T_3\mathbf{c}_3 = T'_1\mathbf{f}_1 + T'_2\mathbf{f}_2 + T'_3\mathbf{f}_3.$$

Then the scalar products of the torques with the unit vectors a, b, c /183 will be:

$$\begin{aligned}
\mathbf{T}\cdot\mathbf{a} &= T_2(1-x^2)^{1/2} + T_3 z = T'_2(1-x^2)^{1/2} + T'_3 x,\\
\mathbf{T}\cdot\mathbf{b} &= T_3 = T'_1\frac{w}{(1-x^2)^{1/2}} + \frac{T'_2(z-xy)}{(1-x^2)^{1/2}} + T'_3 y,\\
\mathbf{T}\cdot\mathbf{c} &= T_1\frac{w}{(1-z^2)^{1/2}} + T_2\frac{(x-yz)}{(1-z^2)^{1/2}} + T_3 y = T'_3.
\end{aligned} \qquad (47)$$

The quantities T_1 and T'_1 appear in (47) with the factor w, which vanishes with averaging over the precession period. Thus, the terms T_1 and T'_1 can be thrown out at once. Specific expressions for T_2, T_3, T'_2 and T'_3 should now be specified for the derivation of the final system of equations. Goldreich does this in a twofold manner. Firstly, he uses the formula of Mac Donald [5], in which only the tidal interaction of the Earth and Moon are taken into account, and the tidal torque is proportional to the lag angle (more precisely, sin 2δ) independently of the angular velocities Ω and ω. His expression is analogous to (2.9), but includes factors taking into account the inclination of the lunar orbit to the terrestrial equator. Secondly, he uses the more complex expression of Kaula [17], who developed the method of G. Darwin. The expression for the tidal potential in it is represented by a Fourier series in linear combinations of the frequencies $\Omega_\oplus$, ω_ζ and $\omega_\odot$. Thus, in addition to the differences in the expressions for the torques according to Mac Donald and Kaula, the solar tides on the Earth are not taken into account in the first place, and are taken into account in the second. It should be noted in connection with the latter difference that the effect of solar tides on the Earth at present as compared to the lunar is secondary: the solar tides dissipate 3 - 5 less energy than the lunar tides. In the past, when the lunar orbit was closer to the Earth, the role of the lunar tides was much greater, whereas the effect of the solar tides has remained unchanged. Thus, neglecting the solar tides in the formulae of Mac Donald is quite valid. The numerical results of Goldreich complete verified this (cf. Figure 2.10, 2.11).

REFERENCE DATA ON THE MOON*

Ratio of the masses of the Earth and Moon	81.3015
Average density	3.34 gm/cm^3
Synodic month (interval between new Moons)	29.530588 days
Sidereal month (interval between the same positions with respect to the stars)	27.321611 days
Inclination of the lunar orbit to the ecliptic	5°8'43"
Inclination of the lunar equator to the ecliptic	1°32'40"
Inclination of the lunar orbit to the terrestrial equator	from 18°5 to 28°5
Average distance of the Moon from the Earth	384,400 km
Diameter of the Moon	3,476 km
Escape velocity from the surface of the Moon	2.38 km/sec
Acceleration of gravity at the surface	162.2 cm/sec^2
Average stellar magnitude of the Moon at full Moon	$-12,^{m7}$
Average albedo	0.07
Temperature at the surface	from 120K to 407K

* Lunar Chart, 1st edition, NASA, March 1970.

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